UNIT VIII

ATOMS AND NUCLEI

Weightage Marks : 06

TOPICS TO BE COVERED

Alpha-particles scattering experiment, Rutherford's model of atom, Bohr Model, energy levels, Hydrogen spectrum. Composition and size of Nucleus, atomic masses, isotopes, isobars; isotones, Radioactivity-alpha, beta and gamma particles/rays and their properties; radioactive decay law.

KEY POINTS

- Gieger-Marsden α-scattering experiment established the existence of nucleus in an atom.

  Bohr's atomic model

  (i) Electrons revolve round the nucleus in certain fixed orbits called stationary orbits.

  (ii) In stationary orbits, the angular momentum of electron is integral multiple of $h/2\pi$.

  (iii) While revolving in stationary orbits, electrons do not radiate energy. The energy is emitted (or absorbed) when electrons jump from higher to lower energy orbits. (or lower to higher energy orbits). The frequency of the emitted radiation is given by $h\nu = E_f - E_i$. An atom can absorb radiations of only those frequencies that it is capable of emitting.

- As a result of the quantisation condition of angular momentum, the electron orbits the nucleus in circular paths of specific radii. For a hydrogen atom it is given by.

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2} \quad \Rightarrow \quad r_n \propto n^2$$

The total energy is also quantised : $E_n = \dfrac{-me^4}{8n2\varepsilon_0^2 h^2} = -13.6\text{eV}/\text{n}^2$

The n = 1 state is called the ground state.

In hydrogen atom, the ground state energy is – 13.6 eV.

- de Broglie's hypothesis that electron have a wavelength $\lambda = h/mv$ gave an explanation for the Bohr's quantised orbits.
- Neutrons and protons are bound in nucleus by short range strong nuclear force. Nuclear force does not distinguish between nucleons.
- The nuclear mass 'M' is always less than the total mass of its constituents. The difference in mass of a nucleus and its constituents is called the **mass defect.**

$$\Delta M = [Zm_p + (A - Z)m_n] - \text{M and } \Delta E_b = (\Delta M)c^2$$

The energy ΔE_b represents the binding energy of the nucleus.

For the mass number ranging from A = 30 to 170 the binding energy per nucleon is nearly constant at about 8MeV per nucleon.

- *Radioactive Decay Law :* The number of atoms of a radioactive sample disintegrating per second at any time is directly proportional to the number of atoms present at that time. Mathematically :

$$\frac{dN}{dt} = -\lambda N \text{ or } N_{(t)} = N_0 e^{-\lambda t}$$

where λ is called decay constant. It is defined as the reciprocal of the time during which the number of atoms of a radioactive substance decreases to 1with of their original number.

- Number of radioactive atoms N in a sample at any time t can be calculated using the formula.

$$N = N_0 \left(\frac{1}{2}\right)^{t/T}$$

Here No = no. of atoms at time t = 0 and T is the half-life of the substance.

Half life : The half life of a radio active substances is defined as the time during which the number of atoms disintegrate to one half of its initial value.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \ln 2 \times \text{mean life} = \frac{.693}{\lambda}$$

Here λ = decay constant= $\frac{1}{\text{mean life}}$.

- Radius r of the nucleus of an atom is proportional to the cube root of its mars number thereby implying that the nuclear density is the same. (Almost) for all substances/nuclei.

- α-decay : ${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q$

  β-decay : ${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + {}_{-1}e^{0} + \bar{\nu} + Q$

  γ-decay : When α or β-decay leave, the nucleus in excited state; the nucleus goes to lower energy state or ground state by the emission of γ-ray(s).

VERY SHORT ANSWER QUESTIONS (I Mark)

1. Define atomic mass unit. Write its energy equivalent in MeV.
2. What was the drawback of Rutherford model of atom?
3. What are the number of electrons and neutrons in singly ionised ${}^{236}_{92}U$ atom?
4. Name the series of hydrogen spectrum which has least wavelength.

*5. Any two protons repel each other, then how is this possible for them to remain together in a nucleus.

6. Define radioactive decay constant.
7. You are given reaction : ${}_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{2}He^{4}$ + 24 MeV. What type of nuclear reaction is this?

8. After losing two electrons, to which particle does a helium atom get transformed into?

9. Write two important inferences drawn from Gieger-Marsden's α-particle scattering experiment.

10. What will be the ratio of the radii of the nuclei of mass number A_1 and A_2?

11. In nuclear reaction ${}^{1}_{1}H \rightarrow {}^{1}_{0}n + {}^{P}_{Q}x$ find P, Q and hence identify X.

12. Binding energies of neutron $\left({}^{2}_{1}H\right)$ and α-particle $({}_2He^4)$ are 1.25 MeV/ nucleon and 7.2 MeV/nucleon respectively. Which nucleus is more stable?

13. α-particles are incident on a thin gold foil. For what angle of deviation will the number of deflected α-particles be minimum?

14. A and B are two isotopes having mass numbers 14 and 16 respectively. If the number of electrons in A is 7, then give the number of neutrons in B.

15. If the amount of a radioactive substance is increased four times then how many times will the n umber of atoms disintegrating per unit time be increased?

16. An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom?

17. Under what conditions of electronic transition will the emitted light be monochromatic?

18. Why does only a slow neutron (.03eV energy) cause the fission in the uranium nucleus and not the fast one?

19. Write the relation for distance of closest approach.

20. In B ohr's a tomic model, t he potential e nergy i s negative and h as a magnitude greater than the kinetic energy, what does this imply?

    Ans. : The electron revolving is bound to the nucleus.

21. Name the physical quantity whose dlimensions are same as Planck's constant. **Ans. :** Angular momentum

22. Define ionisation potential.

23. The ionisation potential of helium atom is 24.6 V. How much energy will be required to ionise it? **Ans. :** 24.6 eV

24. What is the energy possessed by an electron whose principal quantum number is infinite?

25. Write the value of Rydberg constant? **Ans. :** $1.097 \times 10^7\ m^{-1}$

26. Name the spectral series of hydrogen atom which lie in *uv* region.

    Ans. : Lyman Series

27. Name the series of hydrogen spectrum lying in the infra red region.

28. What is the order of velocity of electron in a hydrogen atom in ground state. **Ans. :** $10^6\ ms^{-1}$

29. Write a relation for Paschen series lines of hydrogen spectrum.

    Ans. : $\bar{\nu} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right) n = 4, 5........$

30. Arrange radioactive radiation in the increasing order of penetrating power.

31. Write a relation between average life and decay constant.

32. Write two units for activity of radioactive element and relate them with number of disintegration per second.

33. The half life of a radioactive element A is same as the mean life time of another radioactive element B. Initially, both have same number of atoms. B decay faster than A. Why?

    Ans. : $T_A = \tau_B = 1.44\ T_B \therefore T_A > T_B \therefore \lambda_A < \lambda_B$. Therefore B decay faster than A.

34. Draw the graph showing the distribution of Kinetic energy of electrons emitted during β decay.

35. Compare radii of two nuclei of mass numbers 1 and 27 respectively (**Ans. :** 1 : 3).

36. Define atomic mass unit.

37. Write the energy equivalent of MeV.

38. Which element has highest value of Binding Energy per nucleon.

39. Mention the range of mass number for which the Binding energy curve is almost horizontal.

40. What is the ratio of nuclear densities of the two nuclei having mass numbers in the ratio 1 : 4?

41. Write two important inferences drawn from Rutherford α particle scattering experiment.

42. Draw a graph of number of undecayed nuclei to the time, for a radioactive nuclei. NCERT pg. 447

43. Write an equation to represent α decay.

SHORT ANSWER QUESTIONS (2 Marks)

1. Define distance of the closest approach. An α-particle of kinetic energy 'K' is bombarded on a thin gold foil. The distance of the closet approach is 'r'. What will be the distance of closest approach for an α-particle of double the kinetic energy?

2. Show that nuclear density is independent of the mass number.

3. Which of the following radiations α, β and γ are :
   - (i) similar to x-rays?
   - (ii) easily absorbed by matter
   - (iii) travel with greatest speed?
   - (iv) similar to the nature of cathode rays?

4. Some scientist have predicted that a global nuclear war on earth would be followed by 'Nuclear winter'. What could cause nuclear winter?

5. If the total number ofneutrons and protons in a nuclear reaction is conserved how then is the energy absorbed or evolved in the reaction?

6. In the ground state of hydrogen atom orbital radius is 5.3×10^{-11} m. The atom is excited such that atomic radius becomes 21.2×10^{-11} m. What is the principal quantum number of the excited state of atom?

7. Calculate the percentage of any radioactive substance left undecayed after half of half life.

8. Why is the density of the nucleus more than that of atom?

9. The atom ${}_8O^{16}$ has 8 protons, 8 neutrons and 8 electrons while atom ${}_4Be^8$ has 4 proton, 4 neutrons and 4 electrons, yet the ratio of their atomic masses is not exactly 2. Why?

*10. What is the effect on neutron to proton ratio in a nucleus when β^- particle is emitted? Explain your answer with the help of a suitable nuclear reaction.

11. Why must heavy stable nucleus contain more neutrons than protons?

12. Show that the decay rate R of a sample of radio nuclide at some instant is related to the number of radio active nuclei N at the same instant by the expression $R = -N\lambda$.

13. What is a nuclear fusion reaction? Why is nuclear fusion difficult to carry out for peaceful purpose?

14. Write two characteristic features of nuclear forces which distinguish them from coulomb force.

15. Half life of certain radioactive nuclei is 3 days and its activity is 8 times the 'safe limit'. After how much time will the activity of the radioactive sample reach the 'safe limit'?

16. Derive $mvr = \frac{nh}{2\pi}$ using de Broglie equation.

17. Draw graph of number of scattered particles to scattering angle in Ratherford's experiment.

18. Show that nuclear density is same for all the nuclei.

19. What is the shortest wavelength present in the (i) Paschen series (ii) Balmer series of spectral lines?
Ans. : 820nm, (ii) 365 nm

20. The radius of the inner most electron orbit of a hydrogen atom 0.53 Å. What are the radii of the $n = 2$ and $n = 3$ orbits.

21. The ground state energy of hydrogen atom is –13.6 eV. What are the Kinetic and potential energies of the electron in this state?

22. Write formula of frequency to represent (i) Lyman series (ii) Balmer series.

23. From the relation $R = R_0 A^{1/3}$ where R_0 is a constant and A is the mass number of a nucleus, show that nuclear matter density is nearly constant.

Ans. : Nuclear matter density $= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$

$$= \frac{mA}{\frac{4}{3}\pi R^3} = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$= \frac{m}{\frac{4}{3}\pi R_0^3} \approx 2.3 \times 10^{17} kg / m^3$$

= Constant

24. Find the energy equivalent of one atomic mass unit in joules and then in MeV.

Ans. : $E = \Delta mc^2$ $\Delta m = 1.6605 \times 10^{-27}$ kg

$= 1.6605 \times 10^{-27} \times (3 \times 10^8)^2$

$= 1.4924 \times 10^{-4}\ J$

$$= \frac{1.4924 \times 10^{-10}}{1.6 \times 10^{-19}} eV$$

$= 0.9315 \times 10^9$ eV

= 931.5 MeV

25. Write four properties of nuclear force.

SHORT ANSWER QUESTIONS (3 Marks)

*1. Give one example of a nuclear reaction. Also define the Q-value of the reaction. What does Q > 0 signify?

2. Explain how radio-active nucleus can-emit β-particles even though nuclei do not contain these particles. Hence explain why the mass number of radioactive nuclide does not change during β-decay.

3. Define the term half life period and decay constant. Derive the relation between these terms.

4. State the law of radioactive decay. Deduce the relation $N = N_0 e^{-\lambda t}$, where symbols have their usual meaning.

5. Give the properties of α-particles, β-particles and γ-rays.

6. With the help of one example, explain how the neutron to proton ratio changes during alpha decay of a nucleus.

7. Distinguish between nuclear fusion and fission. Give an example of each.

8. A radioactive isotope decays in the following sequence

$$A \xrightarrow{{}_0n^1} A_1 \xrightarrow{\alpha} A_2.$$

If the mass and atomic numbers of A_2 are 171 and 76 respectively, find mass and atomic number of A and A_1. Which of the three elements are isobars?

9. Obtain a relation for total energy of the electron in terms of orbital radius. Show that total energy is negative of K.E. and half of potential energy.

$$E = -\frac{e^2}{8\pi E_0 r}.$$

10. Draw energy level diagram for hydrogen atom and show the various line spectra originating due to transition between energy levels.

11. The total energy of an electron in the first excited state of the hydrogen atom is about –3.4 eV. What is

(a) the kinetic energy,

(b) the potential energy of the electron?

(c) Which of the answers above would change if the choice of the zero of potential energy in changed to (i) + 0.5 eV (ii) –0.5 eV.

Ans.

(a) When P.E. is chosen to be zero at infinity E = – 3.4 eV, using E = – K.E., the K.E. = + 3.4 eV.

(b) Since P.E. = – 2E, PE = – 6.8 eV.

(c) If the zero of P.E. is chosen differently, K.E. does not change. The P.E. and T.E. of the state, however would alter if a different zero of the P.E. is chosen.

(i) When P.E. at ∞ is + 0.5 eV, P.E. of first excited state will be –3.4 – 0,5 = – 3.9 eV.

(iii) When P.E. at ∞ is + 0.5 eV, P.E. of first excited state will be –3.4 – (–0.5) = –2.9 eV.

12. What is beta decay? Write an equation to represent β^- and β^+ decay. Explain the energy distribution curve is β decay.

13. Using energy level diagram show emission γ rays by $^{60}_{27}Co$ nucleus and subsequent β decay to obtain $^{60}_{28}Ni$. NCERT pg. 457

LONG ANSWER QUESTIONS (5 Marks)

1. State Bohr's postulates. Using these postulates, drive an expression for total energy of an electron in the n[th] orbit of an atom. What does negative of this energy signify?

2. Define binding energy of a nucleus. Draw a curve between mass number and average binding energy per nucleon. On the basis of this curve, explain fusion and fission reactions.

3. State the law of r adioactive disintegration. Hence define disintegration constant and half life period. Establish relation between them.

4. What is meant by nuclear fission and nuclear chain reaction? Outline the conditions necessary for nuclear chain reaction.

5. Briefly explain Rutherford's experiment for scattering of α particle with the help o f a diagram. Write t he c onclusion made an d d raw t he model suggested.

6. State law of radioactive decay obtain relation

   (i) $N = N_0 e^{-\lambda t}$

   (ii) $R = R_0 e^{-\lambda t}$

   where N is number of radioactive nuclei at time t and

   N_0 is number of radioactive nuclei at time t_0 λ is decay constant

   R is rate of decay at any instant t

   R_0 is rate of decay at any time t_0 (initial time).

NUMERICALS

1. The radius of innermost orbit of Hydrogen atom is 5.3×10^{-1} m. What are the radii of $n = 2$ and $n = 3$ orbits.

Ans. : $r_2 = 2.12 \times 10^{-10}$ m, and $r_3 = 4.77 \times 10^{-10}$ m

2. Calculate the radius of the third Bohr orbit of hydrogen atom and energy of electron in that orbit.
Ans. : r_3 = 4775 A° and $E_3 = -2.43 \times 10^{-19}$ J

3. Calculate the longest and shortest wavelength in the Balmer series of Hydrogen atom. Rydberg constant = 1.0987×10^7 m^{-1}. **Ans. :** λ_l = 6563 A°, λ_s = 3646 A°

4. What will be the distance of closest approach of a 5 MeV proton as it approaches a gold nucleus? **Ans. :** 4.55×10^{-14} m

5. A 12.5 MeV alpha – particle approaching a gold nucleus is deflected by 180°. What is the closest distance to which it approaches the nucleus?
Ans. : 1.82×10^{-14} m

6. Determine the speed of the electron in n = 3 orbit of hydrogen atom.
Ans. : 7.29×10^5 ms^{-1}

7. There are $4\sqrt{2} \times 10^6$ radioactive nuclei in a given radio active element. If half life is 20 seconds, how many nuclei will remain after 10 seconds?
Ans. : 4×10^6

8. The half life of a radioactive substance is 5 hours. In how much time will 15/16 of the material decay? **Ans. :** 20 hours

9. At a given instant, there are 25% undecayed radioactive nuclei in a sample. After 10 seconds, the number of undecayed nuclei reduces to 12.5%. Calculate the mean life of nuclei. **Ans. :** 14.43

10. Binding energy of ${}_2He^4$ and ${}_3Li^7$ nuclei are 27.37 MeV and 39.4 MeV respectively. Which of the two nuclei is more stable? Why? **Ans. :** ${}_2He^4$ because its BE/nucleon is greater

11. Find the binding energy and binding energy per nucleon of nucleus ${}_{83}B^{209}$. Given : mass of proton = 1.0078254 u. mass of neutron = 1.008665 u. Mass of ${}_{83}Bi^{209}$ = 208.980388u.
Ans. : 1639.38 MeV and 7.84 MeV/Nucleon

12. Is the fission of iron (${}_{26}Fe^{56}$) into (${}_{13}Al^{28}$) as given below possible?

$${}_{26}Fe^{56} \longrightarrow {}_{13}Al^{28} + {}_{13}Al^{28} + Q$$

Given mass of ${}_{26}Fe^{56}$ = 55.934940 and ${}_{13}Al^{28}$ = 27.98191 U
Ans. : Since Q value comes out negative, so this fission is not possible

13. Find the maximum energy that β-particle may have in the following decay :

$${}_8O^{19} \rightarrow {}_9F^{19} + {}_{-1}e^0 + \bar{\nu}$$

Given

$m\ ({}_8O^{19}) = 19.003576$ a.m.u.

$m\ ({}_9F^{19}) = 18.998403$ a.m.u.

$m\ ({}_{-}e^0) = 0.000549$ a.m.u. **Ans. :** 4.3049 MeV

14. The value of wavelength in the lyman series is given as

$$\lambda = \frac{93.4 n_i^2}{n_i^2 - 1} A^o$$

Calculate the wavelength corresponding to transition from energy level 2, 3 and 4. Does wavelength decreases or increase.

Ans. : $\lambda_{21} = \frac{913.4 \times 2^2}{2^2 - 1} = 1218\ A^o$

$$\lambda_{31} = \frac{913.4 \times 3^2}{3^2 - 1} = 1028\ A^o$$

$$\lambda_{41} = \frac{913.4 \times 4^2}{4^2 - 1} = 974.3\ A^o$$

$\lambda_{41} < \lambda_{31} < \lambda_{21}$

15. The half life of ${}^{238}_{92}u$ undergoing α decay is 4.5×10^9 years what is the activity of 1g. sample of ${}^{238}_{92}u$.

Ans. : $T_{1/2} = 4.5 \times 10^9\ y$

$= 4.5 \times 10^9 \times 3.16 \times 10^7$ s

$= 1.42 \times 10^{17}$ s

1g of ${}^{238}_{92}u$ contains$= \frac{1}{238 \times 10^{-3}} \times 6.025 \times 10^{26}$ atom

$$= 25.3 \times 10^{20} \text{ atoms}$$

$$\therefore \text{ decay rate } = R = \lambda N = \frac{0.693}{T} \times \lambda$$

$$= \frac{0.693 \times 25.3 \times 10^{20}}{1.42 \times 10^{17}} s^{-1}$$

$$= 1.23 \times 10^{4} B_q.$$

ANSWERS

I MARK QUESTIONS

1. An a.m.u. is 1/12 of the mass of a carbon isotope $^{12}C_6$, 1u = 931 MeV.
2. Rutherford's model of atom failed to explain the existence of sharp lines in hydrogen spectrum.
3. No. of electrons = 91, No. of neutrons = 236 – 92 = 144
4. Lyman Series
5. Nuclear force bet ween two protons is 100 times stronger than the electrostatic force.
6. The decay constant of radioactive substance is defined as the reciprocal of that time in which the number of atoms of substance becomes $\frac{1}{e}$ th times the atoms present initially.
7. Fusion reaction.
8. α-particle.
9. (i) Positive charge is concentrated in the nucleus.
   (ii) size of nucleus is very small in comparison to size of atom.
10. $R_1/R_2 = (A_1/A_2)^{1/3}$
11. $P = 0$, $Q = 1$, X is a positron $({}_{+1}e^0)$.

12. Binding energy of ${}_2^4He$ is more than neutron (${}_1H^2$), So, ${}_2He^4$ is more stable.

13. 180°.

14. 9.

15. Four times $\because$ $R = -\lambda N$.

16. 6

17. Only fixed two orbits are involved and therefore single energy value.

18. Slow neutron stays in the nucleus for required optimum time and disturbs the configuration of nucleus.

ANSWERS 2 MARKS QUESTIONS

1. It will be halved.

2. Using the relation $R = R_0A^{1/3}$.

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \Rightarrow \frac{\frac{4\pi}{3}R_1^3}{\frac{4}{3}\pi R_2^3} = \frac{A_1}{A_2} \text{ or } \frac{\frac{4}{3}\pi R_1^3}{A_1} = \frac{\frac{4}{3}\pi R_2^3}{A_2}$$

Hence nuclear density of 1st element = Nuclear density of 2nd element.

3. (i) Similar to x-rays — γ-rays.

   (ii) α-particle.

   (iii) γ-rays.

   (iv) β-particle.

4. Nuclear radioactive waste will hang like a cloud in the earth atmosphere and will absorb sun radiations.

5. The total binding energy of nuclei on two sides need not be equal. The difference in energy appears as the energy released or absorbed.

6. $n = 2$ as $r_n \propto n^2$

7. From relation $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$ when $t = T/2$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{1/2} \quad \text{or} \quad \frac{N}{N_0} = \frac{1}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.9\%.$$

8. Because radius of atom is very large than radius of nucleus.

9. Due to mass defect or different binding energies.

10. Decreases as number of neutrons decreases and number of protons increases.

11. To counter repulsive coulomb forces, strong nuclear force required between neutron–neutron, neutron–proton and proton–proton.

12. $N = N_0 e^{-\lambda t}$ differentiating both sides we get $\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N$ *i.e.,* decay rate

$$R = \frac{dN}{dt} = -\lambda N.$$

13. For fusion, temperature required is from 10^6 to 10^7 K. So, to carry out fusion for peaceful purposes we need some system which can create and bear such a high temperature.

14. Nuclear forces are short range forces (within the nucleus) and do not obey inverse equare law while coulomb forces are long range (infinite) and obey inverse square law.

15. $$\left(\frac{A}{8A}\right) = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

or $$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{t/3}$$

or $$3 = \frac{t}{3}$$

$\Rightarrow$ $$t = 9 \text{ days.}$$