THREE DIMENSIONAL GEOMETRY

KEY POINTS TO REMEMBER

Line in space

- Vector equation of a line through a given point with position vector $\vec{a}$ and parallel to the vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\, \vec{b}$.
- Vector equation of line through two points with position vectors $\vec{a}, \vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$
- Angle θ between lines $\vec{r} = \vec{a_1} + \lambda\, \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\, \vec{b_2}$ is given by **cosθ** = $\dfrac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|}$
- Two lines are **perpendicular** to each other if $\vec{b_1} . \vec{b_2} = 0$.

Plane

- Equation of a plane at a distance of d unit from origin and perpendicular to $\hat{n}$ is $\vec{r} . \hat{n} = d$.
- Equation of plane passing through $\vec{a}$ and normal to $\vec{n}$ **is** $(\vec{r} - \vec{a}) . \vec{n} = 0$.
- Equation of plane passing through three non collinear points with position vectors $\vec{a}, \vec{b}, \vec{c}$ is $(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) X (\vec{c} - \vec{a})] = 0$.
- If three points are collinear, there are infinitely many possible planes passing through them.
- Planes passing through through the intersection of planes $\vec{r} . \vec{n_1} = d_1$ and $\vec{r} . \vec{n_2} = d_2$ is given **by** $(\vec{r} . \vec{n_1} - d_1) + \lambda\, (\vec{r} . \vec{n_2} - d_2) = 0$.
- Angle θ between the two planes $\vec{r} . \vec{n_1} = d_1$ and $\vec{r} . \vec{n_2} = d_2$ is given **by cos θ** = $\dfrac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}|.|\vec{n_2}|}$.
- Two planes are **perpendicular** to each other iff $\vec{n_1} . \vec{n_2} = 0$.
- Two planes are **parallel iff** $\vec{n_1} = \lambda\, \vec{n_2}$ for some scalar $\lambda \neq 0$.
- A line $\vec{r} = \vec{a} + \lambda\, \vec{b}$ is parallel to the plane $\vec{r} . \vec{n} = d$ iff $\vec{b} . \vec{n} = 0$.

ASSIGNMENT

1. Show that the four points (0, -1, -1), (-4, 4, 4), (4, 5, 1) and (3, 9, 4) are coplanar . Find the equation of plane containing them.
2. Find the equation of plane passing through the points (2, 3, 4), (-3, 5, 1) and (4, -1, 2).
3. Find the equation of the plane through the points (2, 1, -1) and (-1, 3, 4) and perpendicular to the plane x - 2y + 4z =10.
4. Show that the following planes are at right angles:
   $\vec{r} . (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$ and $\vec{r} . (-\hat{\imath} - \hat{\jmath} + \hat{k}) = 3$
5. Determine the value of p for which the following planes are perpendicular to each other:
   i. $\vec{r} . (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 7$ and $\vec{r} . (p\hat{\imath} + 2\hat{\jmath} - 7\hat{k}) = 26$
   ii. 2x - 4y + 3z = 5 and x + 2y + pz = 5.
6. Find the equation of the plane passing through the point (-1, -1, 2) and perpendicular to

the planes 3x + 2y - 3z = 1 and 5x - 4y + z = 5.

7. Obtain the equation of the plane passing through the point (1, -3, -2) and perpendicular to the planes x + 2y + 2z = 5 and 3x + 3y + 2z = 8.

8. Find the equation of the plane passing through the origin and perpendicular to each of the planes x + 2y - z = 1 and 3x - 4y + z = 5.

9. Find the equation of the plane passing through the points (1. -1, 2) and (2, -2, 2) and which is perpendicular to the plane 6x - 2y + 2z = 9.

10. Find the equation of the plane through the point (1, 4, -2) and parallel to the plane -2x + y -3z = 7.

11. Find the vector equations of the planes through the intersection of the planes $\vec{r}\,.\,(2\hat{\imath}+6\hat{\jmath})+12=0$ and $\vec{r}\,.\,(3\hat{\imath}-\hat{\jmath}+4\hat{k})=0$ which are at unit distance from the origin.

12.If the line $\vec{r}=(\hat{\imath}-2\hat{\jmath}+\hat{k})+\lambda\,(2\hat{\imath}+\hat{\jmath}+2\hat{k})$ is parallel to the plane $\vec{r}\,.\,(3\hat{\imath}-2\hat{\jmath}+m\hat{k})=14$, find the value of m.

13. Find the equation of the plane through the points (1, 0, -1) ,(3, 2, 2)and parallel to the line $\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{3}$

14. Find the equation of the plane passing through the point (0, 7, -7) & containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$

15. Prove that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ are coplanar. Also, find the plane containing these two lines.

16. Show that the lines given below are coplanar. Also find the equation of the plane containing them.

$\vec{r}=(2\hat{\jmath}-3\hat{k})+\lambda\,(\hat{\imath}+2\hat{\jmath}+3\hat{k})$ and $\vec{r}=(2\hat{\imath}+6\hat{\jmath}+3\hat{k})+\mu\,(2\hat{\imath}+3\hat{\jmath}+4\hat{k})$

17. Find the image of the point (3, -2, 1) in the plane 3x - y + 4z = 2.

18. Find the length and the foot of perpendicular from the point (7, 14, 5) to the plane 2x + 4y - z = 2.

19. Find the reflection of the point (1, 2 -1) in the plane 3x - 5y + 4z = 5.

20. Find the coordinate of the foot of perpendicular drawn from the point (5, 4, 2) to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Hence or otherwise deduce the length of the perpendicular.

21. Find the image of the point with position vector $3\hat{\imath}+\hat{\jmath}+2\hat{k}$ in the plane $\vec{r}\,.\,(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$.

22. Find the length and the foot of the perpendicular from the point (1, 1, 2) to the plane $\vec{r}\,.\,(\hat{\imath}-2\hat{\jmath}+4\hat{k})+5=0$.

23. The cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

24. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Also find the

point of intersection.

25. Find the coordinates of the point where the line through the points (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7.
26. Find the vector and cartesian equation s of the planes containing the two lines $\vec{r}$ =($2\hat{\imath} + \hat{\jmath} - 3\hat{k}$) + λ ($\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$) and $\vec{r}$ =($3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$) + μ ($3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$).
27. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r}$ =($-3\hat{\imath} + \hat{\jmath} + 5\hat{k}$) + λ ($3\hat{\imath} - \hat{\jmath} - 5\hat{k}$). Also show that the plane contains the line $\vec{r}$ =($-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$) + λ ($\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$).
28. Two airships are moving in space along the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ An astronaut wants to move from one ship to another ship when the two airships are closest. What is the least distance between the airships?
29. A bird is located at A (3, 2, 8). She wants to move to the plane 3x + 2y + 6z + 16 = 0 in shortest time. Find the distance she covered.
30. By computing the shortest distance determine whether the following pairs of lines intersect or not:
    $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1} = z = 2$.
31. From a point A (2, 3, 8) in space , a shooter aims to hit the target at P(6, 5, 11). If the line of fire is $\frac{x-2}{4} = \frac{y-3}{2} = \frac{z-8}{3}$, what you think about the success of the shooter?
32. An astronaut at A(7, 14, 5) in space wants to reach a point P on the plane 2x + 4y - z = 2 when AP is least. Find the position of P and also the distance AP travelled by the astronaut.
33. Find the distance of the point (3, 4, 5) from the plane x + y + z = 2 measured parallel to the line 2x = y = z.

Answer key

1. 5x - 7y + 11z +4 = 0 2. x + y -z = 1 3. 18x + 17y + 4z = 49 5. i. 17 ii. 2
6. 5x + 9y + 11z - 8 =0 7. 2x - 4y + 3z -8 = 0 8. x + 2y + 5z = 0 9. x + y - 2z + 4 = 0
10. 2x - y + 3z + 8 = 0 11. $\vec{r}$. ($2\hat{\imath} + \hat{\jmath} + 2\hat{k}$)+ 3 = 0 and $\vec{r}$. ($-\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$) + 3 = 0
12. m = -2 13. 4x - y - 2z - 6 = 0 14. x + y + z = 0 15. x - 2y + z = 0
16. $\vec{r}$. ($\hat{\imath} - 2\hat{\jmath} + \hat{k}$) + 7 = 0 17. (0, -1, -3) 18. (1, 2, 8); $3\sqrt{21}$ units
19. (73/25, -6/5, 39/25) 20. (1, 6, 0) ; $2\sqrt{6}$ units 21. (1, 2, 1)
22. $\frac{13}{12}\sqrt{6}$; (-1/12, 25/12, -2/12) 23. $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$ 24. (-1, -1, -1) 25. (1, -2, 7)
26. $\vec{r}$. ($10\hat{\imath} + 5\hat{\jmath} - 4\hat{k}$) - 37 = 0; 10x + 5y -4z - 37 = 0 27. x - 2y + z = 0
28. $2\sqrt{29}$ units 29. 11 units 30. lines do not intersect
31. he will be successful 32. P(1, 2, 8); $3\sqrt{21}$units 33. 6 units