RELATIONS & FUNCTIONS

KEY POINTS TO REMEMBER

- Relation R from set A to a set B is subset of A x B.
- A x B = {(a, b) : $a \in A, b \in B$}.
- If n(A) = r, n(B) = s from set A to B then n (AxB) = rs & no. of relations = 2^{rs}.
- $\emptyset$ is a relation defined on set A ,called **empty or void relation** .
- $R = AxA$ *is called universal relation.*
- ***Reflexive* relation :** Relation R defined on set A is said to be reflexive iff (a, a) $\in$ R $\forall$ a $\in$ A.
- **Symmetric Relation :** Relation R defined on set A is said to be symmetric if (a, b) $\in$ R, $(b, a) \in R \; \forall \; a, b \in A$.
- **Transitive Relation :** Relation R defined on set A is said to be transitive if (a, b) $\in$ R, $(b, c) \in R$ implies $(a, c) \varepsilon R \; \forall \; a, b \in A$.
- **Equivalence Relation:** A relation R defined on set A is said to be equivalence relation iff it is reflexive, symmetric and transitive.
- **One - One function**: f:A→B is said to be one - one if distinct elements in A has distinct images in B. i.e.$\forall$ $x_1, x_2 \in A$ s.t. $x_1 \neq x_2$ this implies .$f(x_1) \neq f(x_2)$

  OR

  $\forall$ $x_1, x_2 \in A$ s.t. .$f(x_1) = f(x_2)$ implies $x_1 = x_2$
- One-one function is also called **injective function**.
- **Onto function (surjective):** A function f: A→B is said to be an onto iff R_f= B i.e. $\forall$ b $\in$ B , there exist a $\in$ A ,s.t. f(a) =b.
- **Invertible function**: A function f : X → Y is invertible iff it is bijective.
- **Binary Operation**: A binary Operation * defined on set A is a function from A x A→A . * (a,b) is denoted by a * b.
- Binary operation is commutative and associative in nature.
- If * is a binary operation on A , then element e $\in$ A is said to be the identity element iff a * e = e * a $\forall$ a $\in$ A.
- Identity element is unique.
- If * is a binary operation on A , then element b is said to be inverse of a $\in$ A iff a * b = b* a = e
- Inverse of an element if exists, is unique.
- Total number of binary operations on a set consisting of n elements is $\boldsymbol{n^{n^2}}$.
- Total number of binary commutative binary operations on a set consisting of n elements is $\boldsymbol{n^{\frac{n(n-1)}{2}}}$.
- If A and B are two non empty finite sets containing m and n elements respectively, then (i) Number of functions from A to B = $\boldsymbol{n^m}$.

(ii) Number of one - one functions from A to B =

$$\begin{cases} {}^{n}c_{m} \times m!, & if,\ n \geq m \\ 0, & if\ n < m \end{cases}$$

(iii) Number of on - to functions from A to B =

$$\sum_{r=1}^{n} (-1)^{n-r}\, {}^{n}c_{r}\, r^{m}, \quad \text{if } m \geq n$$

(iv) Number of one - one and onto functions from A to B =

$$\begin{cases} n!, & if,\ n = m \\ 0, & if\ n \neq m \end{cases}$$

Total number of binary operations on a set consisting of n elements is $\boldsymbol{n^{n^2}}$.

ASSIGNMENT

Q1. Discuss whether each of the following relations are reflexive, symmetric or transitive:

(i) Relation R_1 on the set R of all real numbers defined as $R_1=\{(a,b): a-b \geq 0\}$

(ii) R= { (a,b) : a = b}

Q2. Let A = {1,2,3,4} and let f = {(1,4),(2,1),(3,3),(4,2)} and g = {(1,3),(2,1),(3,2)(4,4)}, find fog, gof, fof and gog.

Q3. Find gof and fog if (i) $f(x) = |x|$ and $g(x) = |5x - 2|$

(ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

Q4. Let f : R → R and g : R → R be defined as $f(x) = x^2+3x+1$, $g(x) = 2x - 3$. Find fog and gof.

Q5. Let $f(x) = x^2 - 4x + 3 \quad x<3$

$= x - 4 \quad x\geq 3$

$g(x) = x - 3 \quad x<4$

$= x^2 + 2x + 2 \quad x\geq 4$

Describe the function $\frac{f}{g}$.

Q6. $f(x) = x + 1 \quad x\leq 1$

$= 2x + 1 \quad 1<x\leq 2$

$g(x) = x^2 \quad -1<x<2$

$= x + 2 \quad 2\leq x \leq 3$. Find fog and gof.

Q7. Are the following functions invertible in their respective domains? If so find the inverse of each.

(i) $f(x) = \frac{x}{x+1}$ (ii) $f(x) = \sqrt{1-x^2}$

Q8. If the function f : R → R be defined by $f(x) = x^2 + 5x + 9$. Find $f^{-1}(8)$.

Q9. Let S={0,1,2,3,4} and * be an operation on S defined by $a*b = a \oplus_3 b$. Prepare operation table for $\otimes_3$ and $\oplus_3$.

Q10. Let * be a binary operation on A defined by a * b = LCM of (a,b) for all (a,b) ϵ N. Find

(i) 3*4

(ii) 5*7

(iii) 20*16

(iv) 6*9

Q11. Find the number of binary operations on the set {3,4,5}having identity element as 3.

Q12. Find the number of binary operations on the set {6,7,8} having identity element as 6 and $7^{-1} = 7$, $8^{-1} = 8$.

Q13. Check whether the following relations are functions or not. If yes, then find which functions are one one, onto or neither. A = {2,4,6,8} and B = {a,b,c}

(i) f = {(4,a),(6,c),(8,a)}

(ii) g = {(2,a),(4,b),(6,c),(8,a)}

(iii) h = {(2,a),(4,a),(6,b),(8,b)}

Q14. Check whether the following function f : A→B is invertible or not. Describe f^{-1}.
A = {1,2,3,4} B = {2,4,6,8} , f(1)=2, f(2)=4, f(3)=6, f(4)=8.

Q15. Determine whether the function f : S→S where S={1,2,3} and f={(1,2),(2,1),(3,1)} invertible or not.

Q16. Find the number of binary operations that can be defined on a set of 2 elements.

Q17. Find the number of binary operations on set {1,2,3} having e=3 and $4^{-1}=4$, $5^{-1}=4$.

Q18. If R is the relation on N×N → N defined by (a,b) R (c,d) if a + d = b + c. Show that R is an equi - valence relation.

Q19. Let a function f : R→R be defined by f(x) = 3-4x, prove that f is one one and onto.

Q20. Show that f : N→N given by $f(x) = x^2+x+1$ is not invertible.

Q21. Let A = {-1,0,1} and f = {(x,x^2), x ϵ A}. Show that f : A→A is neither one one nor onto.

Q22. Show that f : R→{x ϵ R : -1<x<1} defined by $\frac{x}{1+|x|}$, x ϵ R is one one and onto.

Q23. If f : [0,∞)→[0,∞) and f(x) = $\frac{x}{1+x}$, show that f is one one but not onto.

Q24. Let A = {a,b,c). State the reason for the following where R_1 and R_2 are relations on A.
(i) R_1= {(a,a),(b,b),(a,c)} is not reflexive.
(ii) R_2= {(a,b),(b,a),(a,c),(a,a),(b,b),(c,c)} is not symmetric.

Q25. Let A = N×N and let * be a binary operation on A defined by
{(a,b)*(c,d)=(ad+bc) for all (a,b),(c,d) ϵ N×N}. Show that
(i) * is commutative on A
(ii) * is associative on A
(iii) * has no identity element.

ANSWER KEY

1.(i) Reflexive, Transitive (ii) Reflexive, Symmetric, Transitive

2. fog = {(1,3),(2,4),(3,1),(4,2)}; gof = {(1,4),(2,3),(3,2),(4,1)}; fof = {(1,2),(2,4),(3,3),(4,1)}
gog = {(1,2),(2,3),(3,1),(4,4)}

3. (i) (fog) (x) = $||5x-2||$ (gof) (x) = |5|x| - 2|
(ii) (fog) (x) = 8x (gof) (x) = 2x

4. (fog) (x) = $4x^2 - 6x + 1$ (gof) (x) = $2x^2 + 6x - 1$

5. $\left(\frac{f}{g}\right)(x) = \begin{cases} x-1 & ,x<3 \\ \frac{x-4}{x-3} & ,\ 3 \le x < 4 \\ \frac{x^2+2x+2}{x-4} & ,x \ge 4 \end{cases}$

6. (fog) (x) =

7. (i) Yes, $f^{-1}(x) = \frac{x}{1-x}$ (ii) No

8. $\frac{-5+\sqrt{21}}{2}, \frac{-5-\sqrt{21}}{2}$

10. (i)12 (ii) 35 (iii) 80 (iv) 18

11. 81

12. 9

13. (i) Not a function (ii) Function but not one one (why) (iii) Function but not onto (why)

14. Yes , f^{-1} = { (2,1),(4,2),(6,3),(8,4) }

15. No (why)

16. 16

17. 3