TOPIC 1
RELATIONS & FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XII Ed. 2007
Relations & Functions	(i).Domain , Co domain & Range of a relation	*	(Previous Knowledge)
	(ii).Types of relations	***	Ex 1.1 Q.No- 5,9,12
	(iii).One-one , onto & inverse of a function	***	Ex 1.2 Q.No- 7,9
	(iv).Composition of function	*	Ex 1.3 QNo- 7,9,13
	(v).Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11

SOME IMPORTANT RESULTS/CONCEPTS

** A **relation** R in a set A is called

(i) *reflexive*, if $(a, a) \in R$, for every $a \in A$,

(ii) *symmetric*, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

(iii)*transitive*, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.

** Equivalence Relation : R is equivalence if it is reflexive, symmetric and transitive.

** **Function** :A relation f : A $\rightarrow$ B is said to be a function if every element of A is correlatedto unique element in B.

   * A is domain

   * B is codomain

  * For any *x* element x $\in$ A, function f correlates it to an element in B, which is denoted by f(x)and is called image of *x* under f . Again if y= f(x), then *x* is called as pre-image of y.

  * Range = {f(x) | x $\in$ A }. Range $\subseteq$ Codomain
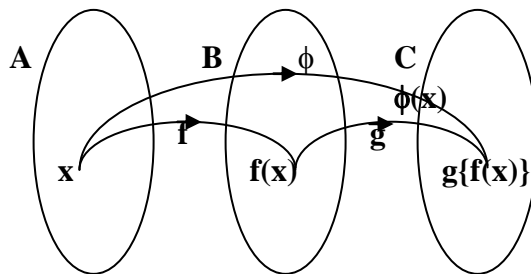
  * The largest possible domain of a function is called domain of definition.

Composite function :

     Let two functions be defined as f : A $\rightarrow$ B and g : B $\rightarrow$ C. Then we can define a function

    ϕ : A $\rightarrow$ C by setting ϕ (x) = g{f(x)} where $x \in$ A, f (x) $\in$ B, g{f(x)} $\in$ C. This function

    ϕ : A $\rightarrow$ C is called the composite function of f and g in that order and we write. $\phi = g \circ f$.

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** **Different type of functions** : Let f : A →B be a function.
 * f is **one to one (injective) mapping**, if any two different elements in A is always correlated to
   different elements in B, i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ or, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
 * f is **many one mapping,** if $\exists$ at least two elements in A such that their images are same.
* f is **onto mapping** (subjective), if each element in B is having at least one preimage.
*f is **into mapping** if range $\subseteq$ codomain.
* f is **bijective mapping** if it is both *one to one and onto*.
** **Binary operation :** A binary operation * on a set A is a function * : A × A → A. We denote
   *(a, b) by a *b.
 * A binary operation '*' on A is a rule that associates with every ordered pair (a, b) of A x A a
unique element a *b.
 * An operation '*' on a is said to be commutative iff a * b = b * a $\forall$ a, b $\in$ A.
 * An operation '*' on a is said to be associative iff (a * b) * c = a * (b * c) $\forall$ a, b, c $\in$ A.
* Given a binary operation * : A × A → A, an element $e \in$ A, if it exists, is called *identity* for the
operation *, if a *$e = a = e$ *a, $\forall a \in$ A.
 * Given a binary operation * : A × A → A with the identity element e in A, an element $a \in$ A is said
to be *invertible* with respect to the operation*, if there exists an element b in A such that
$a * b = e = b * a$ and b is called the *inverse of a* and is denoted by a^{-1}.

ASSIGNMENTS

(i) Domain , Co domain & Range of a relation
LEVEL I
1. If A = {1,2,3,4,5}, write the relation a R b such that a + b = 8, a ,b $\in$ A. Write the domain,
    range & co-domain.
2. Define a relation R on the set **N** of natural numbers by
        R={(x , y) : $y = x$ +7, x is a natural number lesst han 4 ; $x, y \in$ **N**}.
     Write down the domain and the range.

2. *Types of relations*
LEVEL II
1. Let R be the relation in the set N given by R = {(a , b)| a = b – 2 , b > 6}
    Whether the relation is reflexive or not ?justify your answer.
2. Show that the relation R in the set N given by R = {(a , b)| a is divisible by b , a , b $\in$ N}
    is reflexive and transitive but not symmetric.
3. Let R be the relation in the set N given by R = {(a ,b)| a > b} Show that the relation is neither
    reflexive nor symmetric but transitive.
4. Let R be the relation on R defined as (a , b) $\in$ R iff 1+ ab> 0 $\forall$ a,b$\in$ R.
       (a) Show that R is symmetric.
       (b) Show that R is reflexive.
       (c) Show that R is not transitive.
5. Check whether the relation R is reflexive, symmetric and transitive.
      R = { (x , y)| x – 3y = 0} on A ={1, 2, 3……….13, 14}.

LEVEL III

1. Show that the relation R on A, A = { x| x $\in \mathbb{Z}$, $0 \leq x \leq 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

2. Let N be the set of all natural numbers & R be the relation on N $\times$ N defined by
   { (a , b) R (c , d) iff a + d = b + c}. Show that R is an equivalence relation.

3. Show that the relation R in the set A of all polygons as:
   R ={(P$_1$,P$_2$), P$_1$& P$_2$ have the same number of sides} is an equivalence relation. What
   is the set of all elements in A related to the right triangle T with sides 3,4 & 5 ?

4. Show that the relation R on A, A = { x| x $\in \mathbb{Z}$, $0 \leq x \leq 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

5. Let N be the set of all natural numbers & R be the relation on N $\times$ N defined by
   { (a , b) R (c ,d) iff a + d = b + c}. Show that R is an equivalence relation. **[CBSE 2010]**

6. Let A = Set of all triangles in a plane and R is defined by R={(T$_1$,T$_2$) : T$_1$,T$_2$ $\in$ A & T$_1$~T$_2$ }
   Show that the R is equivalence relation. Consider the right angled Δs, T$_1$ with size 3,4,5;
   T$_2$ with size 5,12,13; T$_3$ with side 6,8,10; Which of the pairs are related?

(iii)One-one , onto & inverse of a function

LEVEL I

1. If f(x) = $x^2 - x^{-2}$, then find f(1/x).

2. Show that the function f: R$\rightarrow$R defined by f(x)=x^2 is neither one-one nor onto.

3. Show that the function f: N$\rightarrow$N given by f(x)=2x is one-one but not onto.

4. Show that the signum function f: R$\rightarrow$R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

   is neither one-one nor onto.

5. Let A = {-1,0,1} and B = {0,1}. State whether the function f : A $\rightarrow$ B defined by f(x) = x^2
   isbijective .

6. Let $f(x) = \dfrac{x-1}{x+1}$, x$\neq$ -1,then find f^{-1}(x)

LEVEL II

1. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1,4),(2,5), (3,6)} be a function from A to B.
   State whether f is one-one or not. **[CBSE2011]**

2. If f : R$\rightarrow$R defined as f(x) = $\dfrac{2x-7}{4}$ is an invertible function . Find f^{-1}(x).

3. Write the number of all one-one functions on the set A={a, b, c} to itself.

4. Show that function f :R$\rightarrow$R defined by f(x)= 7– 2x^3 for all x $\in$R is bijective.

5. If f: R$\rightarrow$R is defined by f(x)= $\dfrac{3x+5}{2}$. Find f^{-1}.

LEVEL III

1. Show that the function f: R→R defined by $f(x) = \dfrac{2x-1}{3}$. $x \in$ R is one- one & onto function. Also find the f^{-1}.

2. Consider a function f :R$_+$→[-5, ∞) defined $f(x) = 9x^2 + 6x - 5$. Show that f is invertible & $f^{-1}(y) = \dfrac{\sqrt{y+6}-1}{3}$, where R$_+$ = (0,∞).

3. Consider a function f: R→R given by f(x) = 4x + 3. Show that f is invertible & f^{-1}: R→R with $f^{-1}(y) = \dfrac{y-3}{4}$.

4. Show that f: R→R defined by $f(x) = x^3 + 4$ is one-one, onto. Show that $f^{-1}(x) = (x-4)^{1/3}$.

5. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by
$$f(x) = \left(\dfrac{x-2}{x-3} \right).$$ Show that f is one one onto and hence find f^{-1}. **[CBSE2012]**

6. Show that $f : N \to N$ defined by $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$ is both one one onto.

[CBSE2012]

(iv) *Composition of functions*

LEVEL I

1. If $f(x) = e^{2x}$ and $g(x) = \log \sqrt{x}$, x > 0, find
   (a) (f + g)(x)   (b) (f .g)(x)   (c) f o g (x)   (d) g o f (x).

2. If $f(x) = \dfrac{x-1}{x+1}$, then show that (a) $f\left(\dfrac{1}{x}\right) = -\,f(x)$ (b) $f\left(-\dfrac{1}{x}\right) = \dfrac{-1}{f(x)}$

LEVEL II

1. Let f, g : R→R be defined by f(x)=|x| & g(x) = [x] where [x] denotes the greatest integer function. Find f o g (5/2) & g o f (-√2).

2. Let $f(x) = \dfrac{x-1}{x+1}$. Then find f(f(x))

3. If $y = f(x) = \dfrac{3x+4}{5x-3}$, then find (fof)(x) i.e. f(y)

4. Let f : **R** →**R** be defined as f(x) = 10x +7.Find the function g : **R** →**R** such that g ∘ f (x)= f ∘ g(x) = I$_R$ **[CBSE2011]**

5. If f : **R** →**R** be defined as $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$, then find f ∘ f(x).

[CBSE2010]

6. Let f :R→R& g : R→R be defined as $f(x) = x^2$, g(x) = 2x − 3 . Find fog(x).

(v)Binary Operations

LEVEL I

1. Let * be the binary operation on N given by a*b = LCM of a &b . Find 3*5.
2. Let *be the binary on N given by a*b =HCF of {a ,b}   , a,b∈N. Find 20*16.

3. Let * be a binary operation on the set Q of rational numbers defined as $a * b = \dfrac{ab}{5}$.

   Write the identity of *, if any.
4. If a binary operation '*' on the set of integer Z , is defined by $a * b = a + 3b^2$
   Then find the value of 2 * 4.

LEVEL 2

1. Let A= N×N & * be the binary operation on A defined by (a ,b) * (c ,d) = (a+c, b+d)
   Show that * is   (a) Commutative   (b) Associative (c) Find identity for * on A, if any.

2. Let A = Q×Q. Let * be a binary operation on A defined by (a,b)*(c,d)= (ac , ad+b).
   Find: (i) the identity element of A (ii) the invertible element of A.
3. Examine which of the following is a binary operation

   (i) $a * b = \dfrac{a+b}{2}$;   a, b∈ N   (ii) $a*b = \dfrac{a+b}{2}$ a, b∈ Q

   For binary operation check commutative & associative law.

LEVEL 3

1.Let A= N×N   & * be a binary operation on A defined by   (a , b) × (c , d) = (ac , bd)
   ∀ (a , b),(c , d) ∈N×N   (i) Find (2,3) * (4,1)

    (ii) Find [(2,3)*(4,1)]*(3,5) and (2,3)*[(4,1)* (3,5)] & show they are equal

    (iii) Show that * is commutative & associative on A.

2. Define a binary operation * on the set {0,1,2,3,4,5} as $a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & a + b \geq 6 \end{cases}$

   Show that zero in the identity for this operation & each element of the set is invertible
   with 6 − a being the inverse of a.   **[CBSE2011]**
3. Consider the binary operations ∗ :**R** × **R** → **R**and o : **R** × **R** → **R** defined as a ∗b = |a − b|
   and a o b = a, ∀a, b ∈**R**. Show that ∗is commutative but not associative, o is associative but
   not commutative.   **[CBSE2012]**

Questions for self evaluation

1. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a − b| is even}, is an
   equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the
   elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element
   of {2, 4}.
2. Show that each of the relation R in the set A = {x ∈**Z** : 0 ≤ x ≤ 12}, given by
   R = {(a, b) : |a − b| is a multiple of 4} is an equivalence relation. Find the set of all elements related
   to 1.

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3. Show that the relation R defined in the set A of all triangles as R = {(T$_1$, T$_2$) : T$_1$ is similar to T$_2$}, is equivalence relation. Consider three right angle triangles T$_1$ with sides 3, 4, 5, T$_2$ with sides 5, 12, 13 and T$_3$ with sides 6, 8, 10. Which triangles among T$_1$, T$_2$ and T$_3$ are related?

4. If R$_1$ and R$_2$ are equivalence relations in a set A, show that R$_1 \cap$ R$_2$ is also an equivalence relation.

5. Let A = **R** – {3} and B = **R** – {1}. Consider the function f : A $\rightarrow$ B defined by f (x) $= \left(\dfrac{x-2}{x-3} \right)$.

Is f one-one and onto? Justify your answer.

6. Consider f : **R**+ $\rightarrow$ [– 5, ∞) given by f (x) = 9x^2 + 6x – 5. Show that f is invertible and find f $^{-1}$.

7. On R – {1} a binary operation '$*$' is defined as a $*$ b = a + b – ab. Prove that '$*$' is commutative and associative. Find the identity element for '$*$'. Also prove that every element of R – {1} is invertible.

8. If A = Q $\times$ Q and '$*$' be a binary operation defined by (a, b) $*$ (c, d) = (ac, b + ad),

for (a, b), (c, d) $\in$ A .Then with respect to '$*$' on A
   (i) examine whether '$*$' is commutative & associative
   (i) find the identity element in A,
   (ii) find the invertible elements of A.

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ANSWERS

TOPIC 1 RELATIONS& FUNCTIONS

(i) Domain , Co domain & Range of a relation

LEVEL I

1**.** R = { (3,5),(4,4),(5,3) },Domain = {3, 4, 5}, Range = {3, 4, 5}

2. Domain = {1, 2, 3,}, Range = {8, 9, 10}

(iii).One-one , onto & inverse of a function

LEVEL I

1**.** - f(x)   6. $\dfrac{1+x}{1-x}$

LEVEL II

2**.** $f^{-1}(x) = \dfrac{(4x+7)}{2}$ **3.**6   **5.**$f^{-1}(x) = \dfrac{(2x-5)}{3}$

(iv).Composition of function

LEVEL II

5.$f \circ f(x) = x$ 6 . $4x^2 - 12x + 9$

(v)Binary Operations

LEVEL I

5.   15   2. 4   3. e = 5   4.50

Questions for self evaluation

2. {1, 5, 9}   3. T_1 is related to T_3 6. $f^{-1}(x) = \dfrac{\sqrt{x+6}-1}{3}$

7. e = 0, $a^{-1} = \dfrac{a}{a-1}$ 8. Identity element (1 , 0), Inverse of (a , b) is $\left(\dfrac{1}{a}, \dfrac{-b}{a}\right)$

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