

RELATIONS AND FUNCTIONS

1 Mark

- 1) If $f(x) = \frac{x-1}{x+1}$, ($x \neq -1$), show that $f \circ f^{-1}$ is an identity function.
- 2) If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cdot \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then find the value of $g \circ f(x) = x$
(Ans : 1)
- 3) Let $*$ be a binary operation on the set of real numbers. If $a * b = a+b-ab$, $2 * (3 * x) = -7$, find the value of x .
(Ans : $x=-2$)
- 4) Find the number of One-One functions from a finite set A to A , where $n(A) = P$ (Ans : $P!$)
- 5) Let $A = \{4, 5, 0\}$. Find the number of binary operations that can be defined on A . (Ans: 3^9)
- 6) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = x^2 - 3x + 4$, for all $x \in \mathbb{R}$, find the value of $f^{-1}(2)$
Ans: $\{1, 2\}$
- 7) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = (ax^2 + b)^3$ find the function $g: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(g(x)) = g(f(x))$
Ans: $\left[\frac{x^{\frac{1}{3}} - b}{a} \right]^{\frac{1}{2}}$
- 8) If $f(x) = \frac{5x+3}{4x-5}$ ($x \neq \frac{5}{4}$), find $g(x)$ such that $g \circ f(x) = x$ Ans: $g(x) = \frac{5x+3}{4x-5}$
- 9) If $f(x) = \frac{1+x}{1-x}$, show that $f[f(\tan \theta)] = -\cot \theta$
- 10) Show that $\frac{1}{\sin^3 x} + \cot x + \frac{1}{x^5} + x^3$ is an odd function.
- 11) Let f, g be two functions defined by
 $f(x) = \frac{x}{x+1}, g(x) = \frac{x}{1-x}$, then find $(f \circ g)^{-1}(x)$ Ans: x
- 12) Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, find the value of α such that $f(f(x)) = x$ Ans: $\alpha = -1$

4 Marks / 6 marks

- 13) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$
- 14) If R is a relation on a set A ($A \neq \emptyset$), prove that R is symmetric iff $R^{-1} = R$
- 15) Show that the relation R on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)R(c, d) \Leftrightarrow a+d = b+c$ is an equivalence relation.
- 16) Let $f: \mathbb{R}_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$
- 17) Show that the relation "congruence modulo 2" on the set \mathbb{Z} (set of integers) is an equivalence relation. Also find the equivalence class of 1.

- 18) If the function $f : \mathbb{R} \rightarrow A$ given by $f(x) = \frac{x^2}{x^2 + 1}$ is surjection, find the set A.
 Ans: $A = \text{Range of } f(x) = [0, 1)$
- 19) Let a relation R on the set R of real numbers be defined as $(a, b) \in R \Leftrightarrow 1 + ab > 0$ for all $a, b \in \mathbb{R}$. Show that R is reflexive and symmetric but not transitive.
- 20) Let a relation R on the set R of real numbers defined as $(x, y) \in R \Leftrightarrow x^2 - 4xy + 3y^2 = 0$. Show that R is reflexive but neither symmetric nor transitive.
- 21) Let N denote the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Show that R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$.
- 22) Prove that the inverse of an equivalence relation is an equivalence relation.
- 23) Let $f : A \rightarrow B$ be a given function. A relation R in the set A is given by $R = \{(a, b) \in A \times A : f(a) = f(b)\}$. Check, if R is an equivalence relation. Ans: Yes
- 24) Let f and g be real valued functions, such that $(f \circ g)(x) = \cos x^3$ and $(g \circ f)(x) = \cos^3 x$, find the functions f and g.
 Ans: $f(x) = \cos x$, $g(x) = x^3$
- 25) Define a binary operation * on the set $A = \{0, 1, 2, 3, 4, 5\}$ as
- $$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$
- Show that zero is an identity element for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a.
- 26) Show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x - (-1)^x$ for all $x \in \mathbb{N}$ is a bijection.
- 27) Prove that relation R defined on the set N of natural numbers by $x R y \Leftrightarrow 2x^2 - 3xy + y^2 = 0$ is not symmetric but it is reflexive.
- 28) Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 and R_2 be relations in X given by
 $R_1 = \{(x, y) : x - y \text{ is divisible by } 3\}$ and
 $R_2 = \{(x, y) : \{x, y\} \in \{1, 4, 7\} \text{ or } \{x, y\} \in \{2, 5, 8\} \text{ or } \{x, y\} \in \{3, 6, 9\}\}$. Show that $R_1 = R_2$
- 29) Determine which of the following functions $f : \mathbb{R} \rightarrow \mathbb{R}$ are (a) One - One (b) Onto
 (i) $f(x) = |x| + x$
 (ii) $f(x) = x - [x]$
 (Ans: (i) and (ii) \rightarrow Neither One-One nor Onto)
- 30) On the set N of natural numbers, define the operation * on N by $m * n = \gcd(m, n)$ for all $m, n \in \mathbb{N}$. Show that * is commutative as well as associative.
