TOPIC 1
RELATIONS & FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XII Ed. 2007
Relations & Functions	(i).Domain , Co domain & Range of a relation	*	(Previous Knowledge)
	(ii).Types of relations	***	Ex 1.1 Q.No- 5,9,12
	(iii).One-one , onto & inverse of a function	***	Ex 1.2 Q.No- 7,9
	(iv).Composition of function	*	Ex 1.3 QNo- 7,9,13
	(v).Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11

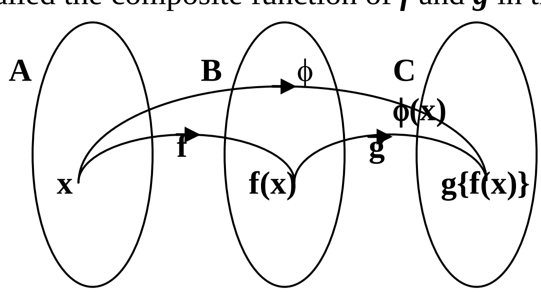
SOME IMPORTANT RESULTS/CONCEPTS

** A **relation** R in a set A is called
(i) *reflexive*, if $(a, a) \in R$, for every $a \in A$,
(ii) *symmetric*, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.
(iii)*transitive*, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.
** Equivalence Relation : R is equivalence if it is reflexive, symmetric and transitive.

** **Function** :A relation $f : A \to B$ is said to be a function if every element of A is correlatedto unique element in B.
* A is domain
* B is codomain
* For any x element $x \in A$, function f correlates it to an element in B, which is denoted by f(x)and is called image of x under f . Again if y= f(x), then x is called as pre-image of y.
* Range = $\{f(x) \mid x \in A\}$. Range $\subseteq$ Codomain
* The largest possible domain of a function is called domain of definition.

Composite function :

Let two functions be defined as $f : A \to B$ and $g : B \to C$. Then we can define a function $\phi : A \to C$ by setting $\phi(x) = g\{f(x)\}$ where $x \in A$, $f(x) \in B$, $g\{f(x)\} \in C$. This function $\phi : A \to C$ is called the composite function of f and g in that order and we write. $\phi = gof$.



** **Different type of functions** : Let ***f*** : A →B be a function.

* ***f*** is **one to one (injective) mapping**, if any two different elements in A is always correlated to different elements in B, i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ or, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

* ***f*** is **many one mapping,** if ∃ at least two elements in A such that their images are same.

* ***f*** is **onto mapping** (subjective), if each element in B is having at least one preimage.

****f*** is **into mapping** if range ⊆ codomain.

* ***f*** is **bijective mapping** if it is both *one to one and onto*.

** **Binary operation :** A binary operation * on a set A is a function * : A × A → A. We denote *(*a*, *b*) by *a* **b*.

* A binary operation '*' on A is a rule that associates with every ordered pair (a, b) of A x A a unique element a *b.

* An operation '*' on a is said to be commutative iff a * b = b * a ∀ a, b ∈ A.

* An operation '*' on a is said to be associative iff (a * b) * c = a * (b * c) ∀ a, b, c ∈ A.

* Given a binary operation * : A × A → A, an element *e* ∈ A, if it exists, is called ***identity*** for the operation *, if *a* **e* = *a* = *e* **a*, ∀*a* ∈ A.

* Given a binary operation * : A × A → A with the identity element *e* in A, an element *a* ∈ A is said to be ***invertible*** with respect to the operation*, if there exists an element *b* in A such that $a * b = e = b * a$ and *b* is called the *inverse of a* and is denoted by a^{-1}.

ASSIGNMENTS

(i) Domain , Co domain & Range of a relation

LEVEL I

1. If A = {1,2,3,4,5}, write the relation a R b such that a + b = 8, a ,b € A. Write the domain, range & co-domain.
2. Define a relation R on the set **N** of natural numbers by

   R={(*x* , *y*) : *y* = *x* +7, *x* is a natural number lesst han 4 ; *x*, *y* ∈ **N**}.

   Write down the domain and the range.

2. *Types of relations*

LEVEL II

1. Let R be the relation in the set N given by R = {(a , b)| a = b – 2 , b > 6}
   Whether the relation is reflexive or not ?justify your answer.
2. Show that the relation R in the set N given by R = {(a , b)| a is divisible by b , a , b ∈ N} is reflexive and transitive but not symmetric.
3. Let R be the relation in the set N given by R = {(a ,b)| a > b} Show that the relation is neither reflexive nor symmetric but transitive.
4. Let R be the relation on R defined as (a , b) ∈R iff 1+ ab> 0 ∀ a,b∈R.
   (a) Show that R is symmetric.
   (b) Show that R is reflexive.
   (c) Show that R is not transitive.
5. Check whether the relation R is reflexive, symmetric and transitive.
   R = { (x , y)| x – 3y = 0} on A ={1, 2, 3……….13, 14}.

LEVEL III

1. Show that the relation R on A ,A = { x| x $\in$ Z , $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

2. Let N be the set of all natural numbers & R be the relation on N $\times$ N defined by
   { (a , b) R (c , d) iff a + d = b + c}. Show that R is an equivalence relation.

3. Show that the relation R in the set A of all polygons as:
   R ={(P_1,P_2), P_1& P_2 have the same number of sides} is an equivalence relation. What is the set of all elements in A related to the right triangle T with sides 3,4 & 5 ?

4. Show that the relation R on A ,A = { x| x $\in$ Z , $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

5. Let N be the set of all natural numbers & R be the relation on N $\times$ N defined by
   { (a , b) R (c ,d) iff a + d = b + c}. Show that R is an equivalence relation. **[CBSE 2010]**

6. Let A = Set of all triangles in a plane and R is defined by R={(T_1,T_2) : T_1,$T_2 \in$ A & T_1~T_2 }
   Show that the R is equivalence relation. Consider the right angled Δs, T_1 with size 3,4,5; T_2 with size 5,12,13; T_3 with side 6,8,10; Which of the pairs are related?

(iii)One-one , onto & inverse of a function

LEVEL I

1. If f(x) = $x^2 - x^{-2}$, then find f(1/x).
2. Show that the function f: R→R defined by f(x)=x^2 is neither one-one nor onto.
3. Show that the function f: N→N given by f(x)=2x is one-one but not onto.
4. Show that the signum function f: R→R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$
   is neither one-one nor onto.
5. Let A = {-1,0,1} and B = {0,1}. State whether the function f : A $\rightarrow$ B defined by f(x) = x^2 isbijective .
6. Let f(x) = $\frac{x-1}{x+1}$, x≠ -1,then find f^{-1}(x)

LEVEL II

1. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1,4),(2,5), (3,6)} be a function from A to B.
   State whether f is one-one or not. **[CBSE2011]**
2. If f : R→R defined as f(x) = $\frac{2x-7}{4}$ is an invertible function . Find f^{-1}(x).
3. Write the number of all one-one functions on the set A={a, b, c} to itself.
4. Show that function f :R→R defined by f(x)= 7– $2x^3$ for all x $\in$ R is bijective.
5. If f: R→R is defined by f(x)= $\frac{3x+5}{2}$. Find f^{-1}.

LEVEL III

1. Show that the function f: R→R defined by $f(x) = \frac{2x-1}{3}$.$x \in R$ is one- one & onto function. Also find the f^{-1}.
2. Consider a function f :R_+→[-5, ∞) defined $f(x) = 9x^2 +6x – 5$. Show that f is invertible & $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$, where $R_+ = (0,\infty)$.
3. Consider a function f: R→R given by f(x) = 4x + 3. Show that f is invertible & f^{-1}: R→R with $f^{-1}(y) = \frac{y-3}{4}$.
4. Show that f: R→R defined by $f(x)= x^3+4$ is one-one, onto. Show that $f^{-1}(x)=(x-4)^{1/3}$.
5. Let $A = R-\{3\}$ and $B = R-\{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one one onto and hence find f^{-1}. **[CBSE2012]**
6. Show that $f : N \to N$ defined by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$ is both one one onto. **[CBSE2012]**

(iv) *Composition of functions*

LEVEL I

1. If $f(x) = e^{2x}$ and $g(x) = \log \sqrt{x}$, x > 0, find
   (a) (f + g)(x) (b) (f .g)(x) (c) f o g (x) (d) g o f (x).
2. If $f(x) = \frac{x-1}{x+1}$, then show that (a) $f\left(\frac{1}{x}\right) = -f(x)$ (b) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

LEVEL II

1. Let f, g : R→R be defined by f(x)=|x| & g(x) = [x] where [x] denotes the greatest integer function. Find f o g (5/2) & g o f (-√2).
2. Let $f(x) = \frac{x-1}{x+1}$. Then find f(f(x))
3. If $y = f(x) = \frac{3x+4}{5x-3}$, then find (fof)(x) i.e. f(y)
4. Let f : **R** →**R** be defined as f(x) = 10x +7.Find the function g : **R**→**R** such that $g \circ f(x) = f \circ g(x) = I_R$ **[CBSE2011]**
5. If f : **R** →**R** be defined as $f(x) = \left(3-x^3\right)^{\frac{1}{3}}$, then find $f \circ f(x)$. **[CBSE2010]**
6. Let f :R→R& g : R→R be defined as $f(x) = x^2$, g(x) = 2x – 3 . Find fog(x).

(v)Binary Operations

LEVEL I

1. Let * be the binary operation on N given by a*b = LCM of a &b . Find 3*5.
2. Let *be the binary on N given by a*b =HCF of {a ,b} , a,b$\in$N. Find 20*16.
3. Let * be a binary operation on the set Q of rational numbers defined as $a * b = \frac{ab}{5}$.

   Write the identity of *, if any.
4. If a binary operation '*' on the set of integer Z , is defined by $a * b = a + 3b^2$
   Then find the value of 2 * 4.

LEVEL 2

1. Let A= N×N & * be the binary operation on A defined by (a ,b) * (c ,d) = (a+c, b+d)
   Show that * is (a) Commutative (b) Associative (c) Find identity for * on A, if any.

2. Let A = Q×Q. Let * be a binary operation on A defined by (a,b)*(c,d)= (ac , ad+b).
   Find: (i) the identity element of A (ii) the invertible element of A.
3. Examine which of the following is a binary operation

   (i) $a * b = \frac{a+b}{2}$; a, b$\in$ N (ii) $a*b = \frac{a+b}{2}$ a, b$\in$ Q

   For binary operation check commutative & associative law.

LEVEL 3

1.Let A= N×N & * be a binary operation on A defined by (a , b) × (c , d) = (ac , bd)
∀ (a , b),(c , d) $\in$N×N (i) Find (2,3) * (4,1)
(ii) Find [(2,3)*(4,1)]*(3,5) and (2,3)*[(4,1)* (3,5)] & show they are equal
(iii) Show that * is commutative & associative on A.

2. Define a binary operation * on the set {0,1,2,3,4,5} as $a * b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & a+b \geq 6 \end{cases}$

Show that zero in the identity for this operation & each element of the set is invertible with 6 – a being the inverse of a. **[CBSE2011]**

3. Consider the binary operations $* : \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ and $o : \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$ defined as $a * b = |a - b|$ and a o b = a, ∀a, b $\in$**R**. Show that $*$is commutative but not associative, o is associative but not commutative. **[CBSE2012]**

Questions for self evaluation

1. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

2. Show that each of the relation R in the set $A = \{x \in \mathbf{Z} : 0 \leq x \leq 12\}$, given by R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

3. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

4. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

5. Let A = **R** – {3} and B = **R** – {1}. Consider the function f : A $\to$ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$.

Is f one-one and onto? Justify your answer.

6. Consider f :**R+** $\to$ [– 5, ∞) given by f (x) = $9x^2 + 6x - 5$. Show that f is invertible and findf^{-1}.

7. On R – {1} a binary operation '$*$' is defined as $a * b = a + b - ab$. Prove that '$*$' is commutative and associative. Find the identity element for '$*$'.Also prove that every element of R – {1} is invertible.

8. If A = $Q \times Q$ and '$*$' be a binary operation defined by $(a, b) * (c, d) = (ac, b + ad)$, for (a, b), (c, d) $\in$ A .Then with respect to '$*$' on A

(i) examine whether '$*$' is commutative & associative

(i) find the identity element in A,

(ii) find the invertible elements of A.

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