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Class 12th

Relations & Functions

Q.1)	Show that the relation R in set Z given by $R\{(a,b) : 2\ divides\ \ a - b\}$ is an Equivalence relation.
Sol.1)	We have, $R = \{(a,b) : 2\ divide\ a - b\}$ Symmetric : let $(a,b) \in R$ $\Rightarrow a - b\ is\ divisible\ by\ 2$ $\Rightarrow a - b = 2\lambda$ ......$\{\lambda \epsilon Z\}$ $\Rightarrow b - a = -2\lambda$ which is also divisible by 2 $\Rightarrow (b,a) \in$ $\therefore$ R is Symmetric Reflexive : for each $a \in Z$ $\Rightarrow a - a = 0$ which is divisible by 2 $\Rightarrow (a,a) \in R$ $\therefore$ R is Reflexive Transitive : let $(a,b) \in R$ and $(b,c) \in R$ $\Rightarrow a - b = 2\lambda$ and $b - c = 2k$ .....$\{\lambda, k \epsilon Z\}$ Now, $a - c = (a - b) + (b - c)$ $\Rightarrow a - c = 2\lambda + 2k$ $\Rightarrow a - c = 2(\lambda + k)$which is also divisible by 2 $\Rightarrow (a,c) \in R$ $\therefore$ R is transitive since R is Symmetric , Reflexive and transitive $\therefore R$ is an Equivalence relation ans.
Q.2)	Show that the relation R in the set A, $A = \{x \in z : 0 \leq x \leq 12\}$ given by $R = \{(a,b) : (a - b)$ is multiple of 4} is an equivalence relation. Find the set of all the elements in set A which are related to 1.
Sol.2)	We have , $R = \{(a,b) : \|a - b\|$ is multiple of 4} Symmetric : let $(a,b) \in R$ $\Rightarrow \|a - b\|$ is multiple of 4 $\Rightarrow \|a - b\| = 4\lambda$ ......$(\lambda \epsilon z)$ $\Rightarrow \|b - a\| = 4\lambda$ which is multiple by 4 $\Rightarrow (b,a) \in R$ $\therefore$ R is Symmetric Reflexive : for each $a \in A$ we have, $\|a - a\| = 0$ which is multiple of 4 $\Rightarrow (a,a) \in R$ $\therefore$ R is Reflexive Transitive :

	let $(a,b) \in R$ and $(b,c) \in R$ $\Rightarrow |a-b| = 4\lambda$ and $|b-c| = 4k$ .....$\{\lambda, k \epsilon Z\}$ $\Rightarrow (a-b) = \pm 4\lambda$ and $(b-c) = -4k$ Now, $(a-c) = (a-b) + (b-c)$ $\Rightarrow (a-c) = \pm 4\lambda \pm 4k$ $\Rightarrow (a-c) = \pm 4(\lambda + k)$ $\Rightarrow |a-c| = |\lambda + k|$ which is multiple of 4 $\Rightarrow (a,c) \in R$ $\therefore$ R is transitive $\therefore$ R is an Equivalence relation The elements which related to 1 are 1, 5, 9 $\therefore$ required set is {1, 5, 9} ans.
Q.3)	Let R be a relation on the set "A" of ordered pairs defined by $(x,y)\,R(u,v)$ if and only if $xv = yu$. Show that R is an equivalence relation.
Sol.3)	Given : A → set of ordered pairs $(x,y)\,R(u,v) \Rightarrow xv = yu$ Symmetric : let $(x,y)\,R\,(u,v)$ $\Rightarrow xv = yu$ (Rough work) $\Rightarrow vx = uy$ $((u,v)\,R(x,y))$ $\Rightarrow uy = vx \Rightarrow (4,v)\,R(x,y)$ $(uy = vx)$ $\therefore$ R is Symmetric Reflexive : for each (x , y) ∈ A $\Rightarrow xy = yx$ {Rough work} $\Rightarrow (x,y)\,R(x,y)$ $\{(x,y)\,R(xy)\}$ $\therefore$ R is Reflexive $\{xy = yx\}$ Transitive : let $(x,y)\;R(u,v)$ and $(u,v)\;R(a,b)$ $\Rightarrow xv = yu$ and $ub = va$ $\Rightarrow xv = yu$ and $v = \frac{ub}{a}$ .....$\{Rough\ (x,y)\,R(a,b)\ ,\ xb = ya\}$ $\Rightarrow x\left(\frac{ub}{a}\right)$= yu $\Rightarrow xb = ya$ $\Rightarrow (x,y)\,R(a,b)$ $\therefore$ R is transitive since R is Symmetric , Reflexive as well as transitive $\therefore$ R is an Equivalence relation ans.
Q.4)	If R_1 and R_2 are equivalence relations in set A , show that $R_1 \cap R_2$ is also on equivalence relation.
Sol.4)	Given :- R_1 and R_2 are equivalence relations Symmetric :

	let $(a,b) \in R_1 \cap R_2$ $\Rightarrow (a,b) \in R_1$ and $(a,b) \in R_2$ $\Rightarrow (b,a) \in R_1$ and $(b,a) \in R_2$ ....{∵ R and R are symmetric relations} $\Rightarrow (b,a) \in R_1 \cap R_2$ $\therefore$ $R_1 \cap R_2$ is Symmetric Reflexive : for each $a \in A$ we have, $(a,a) \in R_1$ and $(a,a) \in R_2$ ....{$R1$ and R_2 are reflexive} $\Rightarrow (a,a) \in R_1 \cap R_2$ $\therefore$ $R_1 \cap R_2$ is Reflexive Transitive : let $(a,b) \in R_1 \cap R_2$ and $(b,c) R_1 \cap R_2$ $\Rightarrow (a,b) \in R_1$ and $(a,b) \in R_2$ and $(b,c) \in R_1$ & $(b,c) \in R_2$ $\Rightarrow (a,b) \in R_1$ and $(b,c) \in R_1$ | (a,b) R and $(b,c) \in R_2$ $\Rightarrow (a,c) \in R$ | $(a,c) \in R_2$ ....{R_1 & R_2 are transitive} $\Rightarrow (a,c) \in R_1 \cap R_2$ $\therefore$ $R_1 \cap R_2$ is transitive since $R_1 \cap R_2$ is Symmetric , Reflexive as well as transitive $\therefore R_1 \cap R_2$ is an Equivalence relation ans.
Q.5)	R is a relation on set N given by $aRb \leftrightarrow b$ is divisible by a; $a.b \in N$ check whether R is Symmetric , reflexive and transitive.
Sol.5)	We have, $aRb \leftrightarrow b$ is divisible by a Symmetric : $2R6 \Rightarrow 6$ is divisible by 2 ....$\left\{\frac{6}{2}=3\right\}$ but $6R2 \Rightarrow 2$ is not div by 6 .....$\left\{\frac{2}{6}=\frac{1}{2}\right\}$ $\therefore$ R is not symmetric Reflexive : for each $a \in N$ a is always divisible by a $\Rightarrow aRa$ $\therefore$ R is Reflexive Transitive : let aRb and bRc $\Rightarrow$ b is divisible by a and c is div by b $\Rightarrow$ $b = a\lambda$ and $c = bk$ ....$\{\lambda, k \epsilon N\}$ $\Rightarrow$ $c = (a\lambda)k$ ....$\{\because b = a\lambda\}$ $\Rightarrow \frac{c}{a} = \lambda k$ clearly c is div by a $\Rightarrow aRc$ $\therefore$ R is transitive ans.
Q.6)	R be relation in P(x) , where x is a non-empty set , given by ARB if only if ACB , where A & B are subsets in $P(x)$. Is R is an equivalence relation on $P(x)$? Justify

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	your answer.
Sol.6)	Let ARB $\Rightarrow\ A \subset B$ then it is not necessary that B is a subset of A i.e. $B \not\subset A$ $\Rightarrow$ B R A $\therefore$ R is not symmetric and hence R is not an equivalence relation eg. $x = \{1,2,3\}$ $P(x) = \{\{1\}\{2\}\{3\}\{1,2\}\{2,3\}\{1,3\}\{1,2,3\}\}$ clearly $\{2\} \subset \{1,2\}$ between$\{1,2\} \subset \{2\}$ $\therefore$ R is not symmetric ans.
Q.7)	Show that the relation R defined in the set A of all triangles as R-{(T_1 , T_2) : T_1 is similar to T_2} is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1 , T_2 and T_3 are related ?
Sol.7)	A $\rightarrow$ set of are triangles $R = \{(T_1, T_2) : T_1 \sim T_2\}$ Symmetric : let $(T_1, T_2) \in R$ $\Rightarrow\ T_1 \sim T_2$ $\Rightarrow\ T_2 \sim T_1$ $\Rightarrow\ (T_2, T_1) \in R$ $\therefore$ R is symmetric Reflexive : for each triangle $T \in A$ $(T, T) \in R$ since every triangle is similar to itself $\therefore$ R is reflexive Transitive : let $(T_1, T_2) \in R$ and $(T_2 \sim T_3) \in R$ $\Rightarrow\ T_1 \sim T_2\ and\ T_2 \sim T_3$ $\Rightarrow\ T_1 \sim T_3$ $\Rightarrow\ T_1, T_3) \in R$ $\therefore$ R is transitive and hence R is an equivalence relation $T_1\ :\ 3, 4, 5$ $T_2\ :\ 5, 12, 13$ $T_3\ :\ 6, 8, 10$ clearly sides of triangles T_1 and T_3 are in equal proportion i.e $\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ $\therefore T_1 \sim T_3$ $\Rightarrow$ T_1 and T_3 are related to each other ans.
Q.8)	Check whether the relation R in R (real no's) define by $R = (a, b): a \leq b^3$is reflexive, symmetric or transitive.

Sol.8)	Symmetric : $(1,2) \in R$ as $1 \leq 2^3$ but $(2,1) \notin R$ since $2 \nleq 13$ $\therefore$ R is not symmetric Reflexive : $\frac{1}{2} \in R$ but $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} \nleq \left(\frac{1}{2}\right)^3$ $\therefore$ R is not reflexive Transitive : $(9,4) \in R$ and $(4,2) \in R$ as $9 \leq 4^3$ and $4 \leq 2^3$ but $(9,2) \notin R$ since $9 \nleq 2^3$ $\therefore$ R is not transitive ans.
Q.9)	Show that the relation R in the set {1,2,3} given by R = {(1 , 1), (2 , 2), (3 , 3), (1 , 2), (2 , 3)} is reflexive neither symmetric nor transitive.
Sol.9)	We have, $A = \{1,2,3\}$ $R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$ since $(1,2) \in R$ but $(2,1) \notin R$ $\therefore$ R is not Symmetric $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$ $\therefore$ R is not transitive for each $a \in A$ $(a, a) \in R$ i.e. $(1,1), (2,2), (3,3) \in R$ $\therefore$ R is reflexive ans.
Q.10)	Determine whether each of the following relations are reflexive, symmetric and transitive (i) Relation in set A = {1,2,3,.......... 13,14} defined by $R = (x, y): 3x - y = 0$. (ii) Relation in N defined as $R = (x, y): y = x + 5; x < 4$. (iii) Relation in set A = {1,2,3,4,5,6} defined as $R = (x, y): y$ is divisible by x. (iv) Relation in Z defined as $R = (x, y): x - y$ is an integer. (v) Relation in R (real nos) defined as $R = (a, b): a \leq b^2$.
Sol.10)	(i) $R = \{(1,3), (2,6), (3,9), (4,12)\}$ ......$(y = 3x)$ clearly $(1,3) \in R$ but $(3,1) \notin R$ $\therefore$ not symmetric $1 \in A$ but $(1,1) \notin R$ $\therefore$ not reflexive

	$(1,3) \in R$ and $(3,9) \in R$ but $(1,9) \notin R$ $\therefore$ not transitive (ii) $R = \{(1,6),(2,7),(3,8)\}$ .....$\{\ldots\ y = x + 5\ and\ x < 4\}$ Do yourself (iii) $R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)\}\ldots\{\ldots y\ is\ divisible\ by\ x\}$ clearly for each $a \in A$ $(a,a) \in R$ i.e. $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \in R$ $\therefore$ R is reflexive $(1,2) \in R$ but $(2,1) \notin R$ since 1 in not divisible by 2 $\therefore$ R is not transitive for each (a,b) and $(b,c) \in R$ clearly $(a,c) \in R$ $\therefore$ R is transitive (iv) Symmetric let $(x,y) \in R$ $\Rightarrow\ x - y = \lambda$ ..... where $\lambda \rightarrow$ integer $\Rightarrow\ y - x = -\lambda$ which is also an integer $\Rightarrow (y,x) \in R$ $\therefore$ R is Symmetric Reflexive and transitive (Do yourself) (v) give same examples as in case of $a \leq b^3$ It is neither symmetric, nor reflexive, nor transitive.