TOPIC 12
PROBABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Probability	(i) Conditional Probability	***	**Article 13.2 and 13.2.1** Solved Examples 1 to 6 Q. Nos 1 and 5 to 15 Ex. 13.1
	(ii)Multiplication theorem on probability	**	**Article 13.3** SolvedExamples 8 & 9 Q. Nos 2, 3, 13 14 & 16 Ex.13.2
	(iii) Independent Events	***	**Article 13.4** Solved Examples 10 to 14 Q. Nos 1, 6, 7, 8 and 11 Ex.13.2
	(iv) Baye's theorem, partition of sample space and Theorem of total probability	***	**Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 ,Q. Nos 1 to 12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex.
	(v) Random variables & probability distribution Mean & variance of random variables	***	**Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1 & 4 to 15 Ex. 13.4
	(vi) Bernoulli,s trials and Binomial Distribution	***	**Articles 13.7, 13.7.1 & 13.7.2** Solved Examples 31 & 32 Q. Nos 1 to 13 Ex.13.5

SOME IMPORTANT RESULTS/CONCEPTS

** **Sample Space and Events** :

The set of all possible outcomes of an experiment is called the sample space of that experiment. It is usually denoted by S. The elements of S are called events and a subset of S is called an event.

ϕ ($\subset$ S) is called an impossible event and

S($\subset$ S) is called a sure event.

** **Probability of an Event.**

(i) If E be the event associated with an experiment, then probability of E, denoted by P(E) is

defined as P(E) $\dfrac{\text{number of outcomes in E}}{\text{number of total outcomes in sample space S}}$

it being assumed that the outcomes of the experiment in reference are equally likely.

(ii) P(sure event or sample space) = P(S) = 1 and P(impossible event) = P(ϕ) = 0.

(iii) If $E_1, E_2, E_3, \dots, E_k$ are mutually exclusive and exhaustive events associated with an experiment

(i.e. if $E_1 \cup E_2 \cup E_3 \cup \dots \cup E_k) = S$ and $E_i \cap E_j = \phi$ for $i, j \in \{1, 2, 3, \dots, k\}$ $i \neq j$), then

$$P(E_1) + P(E_2) + P(E_3) + \dots + P(E_k) = 1.$$

(iv) $P(E) + P(E^C) = 1$

** If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$P(E|F) = \dfrac{P(E \cap F)}{P(F)}$ provided $P(F) \neq 0$

** Multiplication rule of probability : $P(E \cap F) = P(E)\, P(F|E)$
$= P(F)\, P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.

** Independent Events : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.

Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) \,.\, P(F)$.

** Bayes' Theorem : If $E_1, E_2, \dots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \dots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \dots \cup E_n = S$ andA is any event of nonzero probability, then

$$P(Ei|A) = \frac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j).P(A|E_j)} \quad \text{for any } i = 1, 2, 3, \dots, n$$

** The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	...	x_n
P(X) :	p_1	p_2	...	p_n

where, $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$, i = 1, 1, 2,...,

** **Binomial distribution:** The probability of x successes P (X = x) is also denoted by P (x) and is given by $P(x) = {}^nC_x\, q^{n-x} p^x$, $x = 0, 1, \dots, n.$ $(q = 1 - p)$

ASSIGNMENTS

(i) Conditional Probability

LEVEL I

1. If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive events.
2. Find the probability of drawing two white balls in succession from a bag containing 3 red and 5 white balls respectively, the ball first drawn is not replaced.

LEVEL II

1.A dice is thrown twice and sum of numbers appearing is observed to be 6. what is the conditional probability that the number 4 has appeared at least once.

LEVEL III

1.If$P(A) = \dfrac{3}{8}$, $P(B) = \dfrac{1}{2}$ and $P(A \cap B) = \dfrac{1}{2}$, find $P(\overline{A}/\overline{B})$ and $P(\overline{B}/\overline{A})$

(ii)Multiplication theorem on probability

LEVEL II

1.A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find what is the probability that none is red.

2. The probability of A hitting a target is $\frac{3}{7}$ and that of B hitting is $\frac{1}{3}$. They both fire at the target. Find the probability that (i) at least one of them will hit the target, (ii) Only one of them will hit the target.

LEVEL III

1.A class consists of 80 students; 25 of them are girls and 55 are boys, 10 of them are rich and the remaining poor; 20 of them are fair complexioned. what is the probability of selecting a fair complexioned rich girl.

2.Two integers are selected from integers 1 through 11. If the sum is even, find the probability that both the numbers are odd.

(iii) Independent Events

LEVEL I

1. A coin is tossed thrice and all 8 outcomes are equally likely.
E : "The first throw results in head"   F : "The last throw results in tail"
Are the events independent ?

2. Given P(A) = $\frac{1}{4}$, P(B) = $\frac{2}{3}$ and $P(A \cup B) = \frac{3}{4}$. Are the events independent ?

3. If A and B are independent events, Find P(B) if $P(A \cup B) = 0.60$ and P(A) = 0.35.

(iv) Baye's theorem, partition of sample space and Theorem of total probability

LEVEL I

1. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag . Find the probability that the ball drawn is blue in colour.

2. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts . Find the probability of the lost card being a heart.

3. An insurance company insured 2000 scooter and 3000 motorcycles . The probability of an accident involving scooter is 0.01 and that of motorcycle is 0.02 . An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motorcycle.

4. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin.

5. Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy is getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

LEVEL II

1. Find the probability of drawing a one-rupee coin from a purse with two compartments one of which contains 3 fifty-paise coins and 2 one-rupee coins and other contains 2 fifty-paise coins and 3 one-rupee coins.
2. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women.
3. A company has two plants to manufacture bicycles. The first plant manufactures 60 % of the bicycles and the second plant 40 % . Out of that 80 % of the bicycles are rated of standard quality at the first plant and 90 % of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.

LEVEL III

1. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible. What is the probability that the letter has come from
 (i) LONDON (ii) CLIFTON ?
2. A test detection of a particular disease is not fool proof. The test will correctly detect the disease 90 % of the time, but will incorrectly detect the disease 1 % of the time. For a large population of which an estimated 0.2 % have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease.
3. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III , there is one gold and one silver coin.
A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ? **[CBSE 2011]**

(v) Random variables & probability distribution Mean & variance of random variables

LEVEL I

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of spades
2. 4 defective apples are accidentally mixed with 16 good ones. Three apples are drawn at random from the mixed lot. Find the probability distribution of the number of defective apples.
3. A random variable X is specified by the following distribution

X	2	3	4
P(X)	0.3	0.4	0.3

Find the variance of the distribution.

LEVEL III

1. A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice.Find the probability distribution of the number of tails.

2.The sum of mean and variance of a binomial distribution for 5 trials be 1.8. Find the probability distribution.

3. The mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively. Find $P(X \geq 1)$.

(vi) Bernoulli,s trials and Binomial Distribution

LEVEL II

1. If a die is thrown 5 times, what is the chance that an even number will come up exactly 3 times.
2. An experiment succeeds twice as often it fails. Find the probability that in the next six trials, there will be at least 4 success.

3. A pair of dice is thrown 200 times. If getting a sum 9 is considered a success, find the mean and variance of the number of success.

Questions for self evaluation

1. A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5.

2. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of an experiment are performed. Find the probability that the event happens at least once.

3. A football match is either won, draw or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

4. A candidate has to reach the examination center in time. Probability of him going by bus ore scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively. But he reaches in time if he uses other mode of transport. He reached late at the centre. Find the probability that he traveled by bus.

5. Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} kx, & \text{if } x = 0, \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \end{cases}, \text{ k is a + ve constant.}$$

Find the mean and variance of the probability distribution. 1

6. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

7. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

8. Two cards are drawn simultaneously (or successively) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.