

HIGHLY ORDER THINKING QUESTIONS

HOTS - MATRICES / DETERMINANTS

- 1) If $a+b+c=0$ and $\begin{vmatrix} a-x & a & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ then prove that either $x=0$ or $x=\pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$
- 2) If $A = \begin{bmatrix} p & q \\ r & -p \end{bmatrix}$ is such that $A^2 = I$ then find the value of $I - P^2 + qr$
- 3) If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $A + A^T = I$. Find the possible values of θ $\theta = \frac{\pi}{3}$
- 4) Inverse of a square matrix is unique. Give an example to prove it?
- 5) Prove that $\begin{vmatrix} x-3 & x-4 & x-a \\ x-2 & x-3 & x-b \\ x-1 & x-2 & x-c \end{vmatrix} = 0$, where a, b, c are in A.P.
- 6) Using properties of Determinants prove that : $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+ac & c^2 \end{vmatrix} = 4a^2b^2c^2$
- 7) Express $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$ as sum of the symmetric and skew symmetric matrices.
- 8) Prove that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^2$ (Use properties to prove the above)
- 9) Prove the determinant $\begin{vmatrix} x & -\sin\alpha & \cos\alpha \\ \sin\alpha & -x & 1 \\ \cos\alpha & 1 & x \end{vmatrix}$ is independent as α (Ans: Scalar term)
- 10) The total number of elements in a matrix represents a prime number. How many possible orders a matrix can have. 2
- 11) Find the matrix X such that : $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$
- 12) If $f(x) = 3x^2 - 9x + 7$, then for a square matrix A, write $f(A)$ $(3A^2 - 9A + 7I)$
- 13) Prove that $\begin{bmatrix} (a+1) & (a+2) & (a+2) & 1 \\ (a+2) & (a+3) & (a+3) & 1 \\ (a+3) & (a+4) & (a+4) & 1 \end{bmatrix} = -2$

14) If $\begin{bmatrix} \cos^2 A & \cos A \sin A \\ \cos A \sin A & \sin^2 A \end{bmatrix}$, $Y = \begin{bmatrix} \cos^2 B & \cos B \sin B \\ \cos B \sin B & \sin^2 B \end{bmatrix}$

then show that XY is a zero matrix, provided $(A-B)$ is an odd multiple of $\frac{\pi}{2}$

15) Give that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ find the other roots. Hint: Evaluate, find other roots.

16) If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2 + xI = yA$. Find A^{-1}

17) If P and Q are equal matrices of same order such that $PQ = QP$, then prove by induction that $PQ^n = Q^n P$. Further, show that $(PQ)^n = P^n \times Q^n$

18) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5 = ?$ I_3

19) If A and B are two matrices such that $AB = B$ and $BA = A$ then $A^2 + B^2 = ?$ \

20) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ $n \in N$

21) Find the values of a, b, c if the matrix

$$A = \begin{vmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{vmatrix} \text{ satisfy the equation } A^T A = I_3$$

22) Assume X, Y, Z, W and P are matrices of order $(2 \times n), (3 \times k), (n \times 3)$ and $(p \times k)$ respectively, then the restriction on n, k and p so that $py + my$ will be defined are : $(k=3, p=n)$

23) Let A and B be 3×3 matrices such that $A^T = -A$ and $B^T = B$. Then the matrix $\lambda (AB + 3BA)$ is skew symmetric matrix for λ . $(\lambda = 3)$

24) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$, use the result to find A^4 $\begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$

25) For what value of 'K' the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse. $(K = 3/2)$

26) If A is a non-singular matrix of order 3 and $|A| = -4$ Find $|\text{adj } A|$ (16)

27) Given $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that $|A| = -10$. Find $a_{11}c_{11} + a_{12}c_{12}$ (10)

28) If $\begin{vmatrix} x & b & c \\ a & y & c \\ a & b & z \end{vmatrix} = 0$, then find the value of $\frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c}$ (2)

- 29) If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation :

$$x^2 - 6x + 17 = 0 \text{ find } A^{-1} \quad \text{Ans:} \left(\frac{1}{17} \begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix} \right)$$

- 30) Find the matrix x if $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} x \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$

$$X = -\frac{1}{4} \begin{bmatrix} -53 & 18 \\ 25 & -10 \end{bmatrix}$$

- 31) If $P(a) = \begin{bmatrix} \cos a & -\sin a & 0 \\ \sin a & \cos a & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $[P(x)]^{-1} = [P(-x)]$

- 32) If two matrices A^{-1} and B are given how to find $(AB)^{-1}$ verify with an example.

(Find B^{-1} then find $B^{-1} \times A^{-1}$)

- 33) If $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ Verify $(\text{adj}A)^{-1} = \text{adj}(A^{-1})$

- 34) Find the values of a and b such that $A^2 + aI = bA$ where $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ (a=b=8)

- 35) If $P(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ show that $p(\alpha) \times p(\beta) = p(\alpha + \beta)$

- 36) If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$ then prove by Mathematical Induction that : $A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix}$

- 37) If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ find Matrix B such that $AB = I$, Ans : $B = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

- 38) If x, y, z are positive and unequal show that the value of determinant $\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$ is negative.

- 39) If $A + B + C = \pi$, show that $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = 0$

- 40) Find the quadratic function defined by the equation $f(x) = ax^2 + bx + c$
if $f(0) = 6, f(2) = 11, f(-3) = 6$, using determinants.

- 41) If x, y and z all positive, are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P. Prove that $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$

42) If a, b, c are in A.P. then find the value of :
$$\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$$
 (O)

43) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then show that $A^n = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

44) If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ find A^2 Hence find A^6 Ans: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

45) Find x, if $\left[\begin{array}{ccc|c} x-5 & -1 & 0 & 2 \\ 0 & 2 & 1 & 4 \\ 2 & 0 & 3 & 1 \end{array} \right] = 0$ Ans: $x = \pm 4\sqrt{3}$

46) If P and Q are invertible matrices of same order, then show that PQ is also invertible.

47) If the points (2,0), (0,5) and (x,y) are collinear give a relation between x and y.

48) Let $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ find the possible values of x and y, find the values if x = y.

49) If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$, $n \in \mathbb{N}$

50) For any square matrix verify $A(\text{adj } A) = |A|I$
