

Class XII assignment (Chapter 3,4(Matrices and Determinants))

Q.1. Construct a matrix whose general element is given by  $a_{ij}$ :

(i)  $a_{ij} = e^{i-j}$ , order  $2 \times 3$       (ii)  $a_{ij} = \left\lfloor \frac{-3i+j}{2} \right\rfloor$ , order  $3 \times 3$

Q.2. Find the values of  $x, y, z$  and  $w$  which satisfy:

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4w-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2w \end{bmatrix}$$

Q.3. If  $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$  and  $f(x) = x^2 - 6x + 2$ , show that  $f(A) = 0$ .

Q.4. If  $A = \begin{bmatrix} \alpha & \beta \\ 0 & 1 \end{bmatrix}$ ,  $\alpha \neq 0$ , then prove by mathematical induction that

$$A^n = \begin{bmatrix} \alpha^n & \beta \frac{\alpha^n - 1}{\alpha - 1} \\ 0 & 1 \end{bmatrix}, \text{ for all positive integers } n.$$

Q.5. The department of education runs 120 colleges and 15 universities having a strength of 300 lecturers, 100 readers and 50 professors in the universities and 5000 lecturers and 1000 readers in colleges. The monthly salary is Rs. 9000 for professors, and Rs. 8000 for readers and Rs. 5000 for lecturers. Find the monthly salary bill in the colleges and universities.

Q.6. If the matrix  $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$  is skew symmetric, find the values of  $a, b$  and  $c$ .

Q.7. If  $A = [a_{ij}]$  is a square matrix such that  $a_{ij} = i^2 - j^2$ , then write whether  $A$  is symmetric or skew-symmetric.

Q.8. If  $A = [a_{ij}]$  is a skew-symmetric matrix, then write the value of  $\sum_i a_{ij}$ .

Q.9. Using both row as well as column transformation find the inverse of following matrices:

(i)  $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$       (ii)  $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Q.10. If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , verify that  $A^2 - 4A - 5I = 0$ . Hence find  $A^{-1}$ .

Q.11. Find the value of  $x$  for which  $\begin{vmatrix} 2 & x \\ x & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ .

Q.12. If  $A = \begin{bmatrix} -1 & -3 \\ 2 & 1 \end{bmatrix}$ , find the value of determinant of  $A^2 + 2A$ .

Q.13. Prove by using the properties of determinants:

$$(i) \begin{vmatrix} (x+1)(x+2) & x+2 & 1 \\ (x+2)(x+3) & x+3 & 1 \\ (x+3)(x+4) & x+4 & 1 \end{vmatrix} = -2 \quad (ii) \begin{vmatrix} p^2+2p & 2p+1 & 1 \\ 2p+1 & p+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (p-1)^3$$

$$(iii) \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

Q.13. Solve the following equations:

$$(i) \begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0 \quad (ii) \begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0$$

Q.14. If A is a square matrix of order 3X3 with  $|A|=3$  then find:

$$(i) |5A| \quad (ii) |A \cdot adj A| \quad (iii) |AB| \text{ if } |B|=3 \\ (iv) |adj A| \quad (v) |2AB| \quad (vi) \text{ find } k \text{ if } |4A|=k|A| \quad (vii) |A^T|$$

$$Q.15. \text{ If } A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \text{ then find } (AB)^{-1}.$$

Q.16. Find all integers  $\lambda$  for which the following system of equation is consistent:  $x+2y-3z=1$ ,  $2x-\lambda y-3z=2$ ,  $x+2y+\lambda z=3$ .

$$Q.17. \text{ If } A = \begin{bmatrix} 4 & 1 & 2 \\ -5 & -3 & 3 \\ -11 & 1 & -7 \end{bmatrix}, \text{ find } A^{-1} \text{ and hence solve the following system of}$$

equations:  $4x-5y-11z=12$ ,  $x-3y+z=1$ ,  $2x+3y-7=2$ .

$$Q.18. \text{ Find the product of } \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, \text{ using the product solve the}$$

following system of equations:  $x-y=3$ ,  $2x+3y+4z=17$ ,  $y+2z=7$

Q.19. Find the quadratic function defined by  $f(x) = ax^2+bx+c$ , if  $f(0) = 6$ ,  $f(2) = 11$  and  $f(-3) = 6$ , using matrices.

Q.20. Sum of three numbers is 20. If we multiply the first by 2, add the second and subtract the third we get 23. If we multiply first by 3 and add second and third to it we get 46. Find the numbers.