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## MATRICES

## KEY POINTS TO REMEMBER

$>$ Matrix: A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called elements of the matrix.
$>$ Order of a matrix :A matrix having ' m ' rows \& ' n ' columns is called matrix of order mx n.
$>$ Zero Matrix: A matrix having all the elements zero is called zero matrix or null matrix.
$>$ Diagonal Matrix: A square matrix is called a diagonal matrix if all it's non diagonal elements are zero.
$>$ Scalar Matrix: A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.
> Identity Matrix: A scalar matrix in which each diagonal element is 1 ,is called an identity matrix or a unit matrix. It is denoted by I.
$I=\left[a_{i j}\right]_{n \times n}$ where $a_{i j}=\begin{array}{cc}1 \quad \text { if } i=j \\ 0 & \text { if } \mathrm{i} \neq \mathrm{j}\end{array}$
$>$ Transpose of a Matrix: If $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right]_{\mathrm{mxn}}$ be an $\mathrm{m} \times \mathrm{n}$ matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. Transpose of A is denoted by $\mathrm{A}^{\prime}$ or $\mathrm{A}^{\mathrm{T}}$.

Properties of the transpose of a matrix.

1) $\left(A^{\prime}\right)^{\prime}=A$
2) $(A+B)^{\prime}=A^{\prime}+B^{\prime}$
3) $(\mathrm{kA})^{\prime}=k A^{\prime}, k$ is a scalar.
4) $(\mathrm{AB})^{\prime}=\mathrm{B}^{\prime} \mathrm{A}^{\prime}$
$>$ Symmetric Matrix: A square matrix $\mathrm{A}=\left[\mathrm{a}_{\mathrm{ij}}\right]$ is symmetric if $\mathrm{a}_{\mathrm{ij}}=\mathrm{a}_{\mathrm{ji}} \forall \mathrm{i}, \mathrm{j}$. Also a square matrix A is symmetric if $\mathrm{A}^{\prime}=\mathrm{A}$.
$>$ Skew Symmetric Matrix: A square matrix $A=\left[a_{i j}\right]$ is skew symmetric if $\mathrm{a}_{\mathrm{ij}}=-\mathrm{a}_{\mathrm{ji}} \forall \mathrm{i}, \mathrm{j}$. Also a square matrix A is symmetric if $\mathrm{A}^{\prime}=-\mathrm{A}$.

## ASSIGNMENT

1. If $A=\operatorname{diag}(1,-1,2)$ and $B=\operatorname{diag}(2,3,-1)$ find $A+B, 3 A+4 B$.
2. Find a matrix $X$ such that $2 A+B+X=0$ where $A=\left[\begin{array}{cc}-1 & 2 \\ 3 & 4\end{array}\right] B=\left[\begin{array}{cc}3 & -2 \\ 1 & 5\end{array}\right]$


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4. If $\mathrm{A}=\left[\begin{array}{ll}9 & 1 \\ 7 & 8\end{array}\right] \mathrm{B}=\left[\begin{array}{cc}1 & 5 \\ 7 & 12\end{array}\right]$ find a matrix C such that $5 \mathrm{~A}+3 \mathrm{~B}+2 \mathrm{C}$ is a null matrix.
5. In a certain city there are 30 colleges. Each college has 15 peons, 6 clerks, 1 typist and

1 section officer. Express given information as a column matrix. Using scalar multiplication
find the total no. of posts of each kind in all the colleges.
.6. If A, B, C are three matrices such that $\mathrm{A}=\left[\begin{array}{lll}\boxed{x} & y & z\end{array}\right]$

$$
\mathrm{B}=\left[\begin{array}{lll}
a & h & g \\
h & b & f \\
g & f & c
\end{array}\right] \quad \mathrm{C}=\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] \text { find } \mathrm{ABC}
$$

7. If $A=\left[\begin{array}{cc}1 & -1 \\ 2 & 3\end{array}\right] \quad B=\left[\begin{array}{ll}2 & 1 \\ 1 & 0\end{array}\right] \quad$ Is $(A+B)^{2}=A^{2}+2 A B+B^{2}$
8. If $A=\left[\begin{array}{cc}2 & 3 \\ -1 & 2\end{array}\right]$ and $f(x)=x^{2}-4 x+7$. Show that $f(A)=0$. Use this result to find $A^{5}$.
9. If $\mathrm{A}_{\alpha}=\left[\begin{array}{cc}\cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha\end{array}\right]$ prove that
i) $\mathrm{A}_{\alpha} \mathrm{A}_{\beta}=\mathrm{A}_{\alpha+\beta}$
ii) $\left(\mathrm{A}_{\alpha}\right)^{\mathrm{n}}=\left[\begin{array}{cc}\cos n \alpha & \sin n \alpha \\ \sin n \alpha & \cos n \alpha\end{array}\right]$
10. If $A$ and $B$ are square matrices of order $n$ then prove that $A$ and $B$ will commute iff
( $\mathrm{A}-\lambda \mathrm{i}$ ) and ( $\mathrm{B}-\lambda \mathrm{i}$ ) commute for every scalar $\lambda$.
11. Give an example of 3 matrices $\mathrm{A}, \mathrm{B}, \mathrm{C}$ such that $\mathrm{AB}=\mathrm{AC}$ but $\mathrm{B} \neq \mathrm{C}$.
12. A matrix $X$ has $(a+b)$ rows and $(a+2)$ columns while the matrix $Y$ has $(b+1)$ rows and
$(a+3)$ columns. Both matrices XY and YX exists. Find a and b. Can you say XY and YX are of the same type? Are they equal?
13. Show that the matrix $A=\left[\begin{array}{ll}2 & 3 \\ 1 & 2\end{array}\right]$ is a root of the equation $A^{2}-4 A+I=0$
14. If $\mathrm{A}=\left[\begin{array}{ll}a & b \\ 0 & 1\end{array}\right]$ Prove that $\mathrm{A}^{\mathrm{n}}=\left[\begin{array}{cc}a^{n} & \frac{b\left(a^{n}-1\right)}{a-1} \\ 0 & 1\end{array}\right]$
for every positive integer n .
15. If $A=\operatorname{diag}(a, b, c)$,show that $A^{n}=\operatorname{diag}\left(a^{n}, b^{n}, c^{n}\right)$, for all positive integers $n$.

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16. Give examples of matrices
i) $A$ and $B$ such that $A B \neq B A$.
ii) A and B such that $\mathrm{AB}=0$, but $\mathrm{A} \neq 0, \mathrm{~B} \neq 0$.
iii) A and B such that $\mathrm{AB}=0$, but $\mathrm{BA} \neq 0$.
iv) $\mathrm{A}, \mathrm{B}$ and C such that $\mathrm{AB}=\mathrm{AC}$ but $\mathrm{B} \neq \mathrm{C}, \mathrm{A} \neq 0$
17. If $A$ and $B$ are square matrices of the same order, explain why in general
i) $(A+B)^{2} \neq \mathrm{A}^{2}+2 \mathrm{AB}+\mathrm{B}^{2}$
ii) $(\mathrm{A}-\mathrm{B})^{2} \neq \mathrm{A}^{2}-2 \mathrm{AB}+\mathrm{B}^{2}$
iii) $(\mathrm{A}+\mathrm{B})(\mathrm{A}-\mathrm{B}) \neq \mathrm{A}^{2}-\mathrm{B}^{2}$
18. Three shopkeepers $\mathrm{A}, \mathrm{B}$ and C go to a store to buy stationary. 'A' purchases 12
dozen notebooks, 5 dozen pens and 6 dozen pencils . 'B' purchases 10 dozen notebooks , 6 dozen pens and 7 dozen pencils .'C' purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs Rs. 1.25 and a pencil costs 35 paise . Use matrix multiplication to calculate each shopkeeper`s bill.
19. Let $A$ be a square matrix , then prove that $\left(A A^{T}\right)$ and $\left(A^{T} A\right)$ are symmetric matrices .
20. Show that all positive integral powers of a symmetric matrix are symmetric .
21. Out of the given matrices choose that matrix which is a scalar matrix
a) $\left[\begin{array}{ll}0 & 0 \\ 0 & 0\end{array}\right]$
b) $\left[\begin{array}{lll}0 & 0 & 0 \\ 0 & 0 & 0\end{array}\right]$ c) $\left[\begin{array}{ll}0 & 0 \\ 0 & 0 \\ 0 & 0\end{array}\right]$ d) $\left[\begin{array}{l}0 \\ 0 \\ 0\end{array}\right]$
22. If the matrix $\mathrm{A}=\left[\begin{array}{ccc}5 & 2 & x \\ y & z & -3 \\ 4 & t & -7\end{array}\right]$ is a symmetric matrix , find $\mathrm{x}, \mathrm{y}, \mathrm{z}$ and t .
23. If $A=\left[\begin{array}{lll}1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right] \quad B=\left[\begin{array}{ccc}1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$. $A B=I$, find $x+y$.

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## ANSWERS:

$3 \cdot x=1,2 y=1 \pm \sqrt{10}$
Section officers $=30$
Rs. 281.40
4. $\left[\begin{array}{ll}-24 & -10 \\ -28 & -38\end{array}\right]$ 5. Peons $=450$, Clerks $=180$, Typist $=30$,
6. $a x^{2}+2 h x y+b y^{2}+c z^{2}+2 f y z+2 g z x$ 18. Rs. 157.80 , Rs. 167.40,

