

MATRICES

KEY POINTS TO REMEMBER

- **Matrix:** A matrix is an ordered rectangular array of numbers or functions . The numbers or functions are called elements of the matrix.
- **Order of a matrix :**A matrix having 'm' rows & 'n' columns is called matrix of order m x n.
- **Zero Matrix:** A matrix having all the elements zero is called zero matrix or null matrix.
- **Diagonal Matrix:** A square matrix is called a diagonal matrix if all it's non diagonal elements are zero.
- **Scalar Matrix:** A diagonal matrix in which all diagonal elements are equal is called a scalar matrix.
- **Identity Matrix:** A scalar matrix in which each diagonal element is 1, is called an identity matrix or a unit matrix. It is denoted by I.

$$I = [a_{ij}]_{n \times n} \text{ where } a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$
- **Transpose of a Matrix:** If $A = [a_{ij}]_{m \times n}$ be an m x n matrix then the matrix obtained by interchanging the rows and columns of A is called the transpose of the matrix. Transpose of A is denoted by A' or A^T .

Properties of the transpose of a matrix.

- 1) $(A')' = A$
 - 2) $(A + B)' = A' + B'$
 - 3) $(kA)' = kA'$, k is a scalar.
 - 4) $(AB)' = B' A'$
- **Symmetric Matrix:** A square matrix $A = [a_{ij}]$ is symmetric if $a_{ij} = a_{ji} \forall i, j$. Also a square matrix A is symmetric if $A' = A$.
 - **Skew Symmetric Matrix:** A square matrix $A = [a_{ij}]$ is skew symmetric if $a_{ij} = - a_{ji} \forall i, j$. Also a square matrix A is symmetric if $A' = -A$.

ASSIGNMENT

1. If $A = \text{diag}(1,-1,2)$ and $B = \text{diag}(2,3,-1)$ find $A + B, 3A + 4B$.
2. Find a matrix X such that $2A+B+X=0$ where $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$
3. Solve the matrix equation $\begin{bmatrix} x^2 & - \\ 2 & \end{bmatrix} + \begin{bmatrix} 3x & \\ & 2y \end{bmatrix} = \begin{bmatrix} -2 & \\ & 2 \end{bmatrix}$

4. If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ find a matrix C such that $5A+3B+2C$ is a null matrix.

5. In a certain city there are 30 colleges. Each college has 15 peons, 6 clerks, 1 typist and 1 section officer. Express given information as a column matrix. Using scalar multiplication find the total no. of posts of each kind in all the colleges.

6. If A, B, C are three matrices such that $A = \begin{bmatrix} x & y & z \end{bmatrix}$

$$B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \quad C = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ find } ABC$$

7. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ Is $(A+B)^2 = A^2+2AB+B^2$

8. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use this result to find A^5 .

9. If $A_\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ prove that

i) $A_\alpha A_\beta = A_{\alpha+\beta}$

ii) $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{bmatrix}$

10. If A and B are square matrices of order n then prove that A and B will commute iff $(A - \lambda I)$ and $(B - \lambda I)$ commute for every scalar λ .

11. Give an example of 3 matrices A, B, C such that $AB = AC$ but $B \neq C$.

12. A matrix X has $(a+b)$ rows and $(a+2)$ columns while the matrix Y has $(b+1)$ rows and $(a+3)$ columns. Both matrices XY and YX exists. Find a and b . Can you say XY and YX are of the same type? Are they equal?

13. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ is a root of the equation $A^2 - 4A + I = 0$

14. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ Prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$

for every positive integer n .

15. If $A = \text{diag}(a, b, c)$, show that $A^n = \text{diag}(a^n, b^n, c^n)$, for all positive integers n .

16. Give examples of matrices

i) A and B such that $AB \neq BA$.

ii) A and B such that $AB = 0$, but $A \neq 0, B \neq 0$.

iii) A and B such that $AB = 0$, but $BA \neq 0$.

iv) A, B and C such that $AB = AC$ but $B \neq C, A \neq 0$

17. If A and B are square matrices of the same order, explain why in general

i) $(A+B)^2 \neq A^2 + 2AB + B^2$

ii) $(A-B)^2 \neq A^2 - 2AB + B^2$

iii) $(A+B)(A-B) \neq A^2 - B^2$

18. Three shopkeepers A, B and C go to a store to buy stationary. 'A' purchases 12

dozen notebooks, 5 dozen pens and 6 dozen pencils. 'B' purchases 10 dozen notebooks

, 6 dozen pens and 7 dozen pencils. 'C' purchases 11 dozen notebooks, 13 dozen pens

and 8 dozen pencils. A notebook costs 40 paise, a pen costs Rs. 1.25 and a pencil costs

35 paise. Use matrix multiplication to calculate each shopkeeper's bill.

19. Let A be a square matrix, then prove that (AA^T) and $(A^T A)$ are symmetric matrices.

20. Show that all positive integral powers of a symmetric matrix are symmetric.

21. Out of the given matrices choose that matrix which is a scalar matrix

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

22. If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix, find x, y, z and t.

23. If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $AB = I$, find x+y.

ANSWERS:

3. $x=1,2$ $y = 1 \pm \sqrt{10}$

4. $\begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

5. Peons = 450, Clerks = 180 , Typist = 30 ,

Section officers = 30

6. $ax^2 + 2hxy + by^2 + cz^2 + 2fyz + 2gzx$

18. Rs. 157.80, Rs. 167.40,

Rs. 281.40

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