TOPIC 11
LINEAR PROGRAMMING
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Linear Programming	(i) LPP and its Mathematical Formulation	**	**Articles 12.2 and 12.2.1**
	(ii) Graphical method of solving LPP (bounded and unbounded solutions)	**	**Article 12.2.2** Solved Ex. 1 to 5 Q. Nos 5 to 8 Ex.12.1
	(iii) Diet Problem	***	Q. Nos 1, 2 and 9 Ex. 12.2 Solved Ex. 9 Q. Nos 2 and 3 Misc. Ex. Solved Ex. 8 Q. Nos 3,4,5,6,7 of Ex.
	(iv) Manufacturing Problem	***	12.2 Solved Ex.10 Q. Nos 4 & 10 Misc. Ex.
	(v) Allocation Problem	**	Solved Example 7Q. No 10 Ex.12.2, Q. No 5 & 8 Misc. Ex.
	(vi) Transportation Problem	*	Solved Ex.11 Q. Nos 6 & 7 Misc. Ex.
	(vii) Miscellaneous Problems	**	Q. No 8 Ex. 12.2

SOME IMPORTANT RESULTS/CONCEPTS

** Solving linear programming problem using **Corner Point Method**. The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function Z = $ax + by$ at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is **bounded**, M and m are the maximum and minimum values of Z.
   (ii) In case, the feasible region is **unbounded**, we have:
4. (a) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.
   (b) Similarly, m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

ASSIGNMENTS

(i) LPP and its Mathematical Formulation

LEVEL I

1. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem.

(ii) Graphical method of solving LPP (bounded and unbounded solutions)

LEVEL I

Solve the following Linear Programming Problems graphically:

1.Minimise $Z = -3x + 4y$subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.

2.Maximise $Z = 5x + 3y$subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

3.Minimise $Z = 3x + 5y$ suchthat $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

(iii) Diet Problem

LEVEL II

1.A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1,400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of mineral and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost? Also find the least cost.

2. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs. 10 per kg and rice Rs. 20 per kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 gm and 200 gm respectively. In what quantities, should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of protein and carbohydrates at minimum cost ?

(iv) Manufacturing Problem

LEVEL II

1.A company manufactures two articles A and B. There are two departments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs. 6 for each unit of A and Rs. 8 for each unit of B, find the number of units of A and B to be produced per week in order to have maximum profit.

2. A company sells two different produces A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that for B is 125. Profit on each unit of A is Rs. 20 and that on B is Rs. 15. How many units of A and B should be produced to maximize the profit? Solve it graphically

LEVEL III

1.A manufacture makes two types of cups, A and B. Three machines are required to manufacture the cups and the time in minutes required by each is as given below:

Type of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise, and on B it is 50 paise, show that the 15 cups of type A and 30 cups of type B should be manufactured per day to get the maximum profit.

(v) Allocation Problem

LEVEL II

1. Ramesh wants to invest at most Rs. 70,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 10,000 in Bond A and at least Rs. 30,000 in Bond B. If the rate of interest on bond A is 8 % per annum and the rate of interest on bond B is 10 % per annum , how much money should he invest to earn maximum yearly income ? Find also his maximum yearly income.

2. An oil company requires 12,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimize the cost of requirement?

LEVEL III

1. An aeroplane can carry a maximum of 250 passengers. A profit of Rs 500 is made on each executive class ticket and a profit of Rs 350 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

(vi) Transportation Problem

LEVEL III

1. A medicine company has factories at two places A and B . From these places, supply is to be made to each of its three agencies P, Q and R. The monthly requirement of these agencies are respectively 40, 40 and 50 packets of the medicines, While the production capacity of the factories at A and B are 60 and 70 packets are respectively. The transportation cost per packet from these factories to the agencies are given:

Transportation cost per packet (in Rs.)		
From / To	**A**	**B**
P	**5**	**4**
Q	**4**	**2**
R	**3**	**5**

How many packets from each factory be transported to each agency so that the cost of transportation is minimum ? Also find the minimum cost.

Questions for self evaluation

1. Solve the following linear programming problem graphically : Maximize z = x – 7y +190 subject to the constraints $x + y \leq 8$, $x \leq 5$, $y \leq 5$, $x + y \geq 4$, $x \geq 0$, $y \geq 0$.

2. Solve the following linear programming problem graphically : Maximize z = 3x + 5y subject to the constraints $x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$.

3. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains, 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs. 5 per kilogram and rice costs Rs. 4 per kilogram.

4. A shopkeeper deals only in two items — tables and chairs. He has Rs. 6,000 to invest and a space to store at most 20 pieces. A table costs him Rs. 400 and a chair Rs. 250. He can sell a table at a profit of Rs. 25 and a chair at a profit of Rs. 40. Assume that he can sell all items that he buys. Using linear programming formulate the problem for maximum profit and solve it graphically.

5. A small firm manufactures items A and B. The total number of items A and B it can manufacture a day is at most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs. 300 and one unit of item B be Rs. 160, how many of each type of item be produced to maximize the profit ? Solve the problem graphically.

6. A chemist requires 10, 12 and 12 units of chemicals A, B and C respectively for his analysis. A liquid product contains 5, 2, and 1 units of A, B and C respectively and it costs Rs. 3 per jar. A dry product contains 1, 2, and 4 units of A, B and C per carton and costs Rs. 2 per carton. How many of each should he purchase in order to minimize the cost and meet the requirement ?

7. A person wants to invest at most Rs. 18,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 4,000 in Bond A and at least Rs. 5,000 in Bond B. If the rate of interest on bond A is 9 % per annum and the rate of interest on bond B is 11 % per annum , how much money should he invest to earn maximum yearly income ?

8. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to to stitch at least 60 shirts and 32 pants at a minimum labourcost.