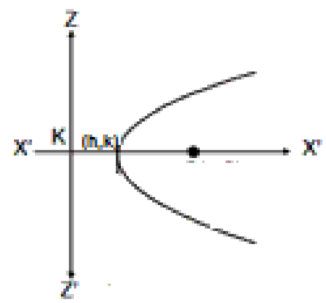
	Class 12 Linear Differential Equation ***Class 12th***
Q.1)	Find the D.E. of all the parabolas with latus rectum $4a$ and whose axes are parallel to $x-axis$.
Sol.1)	Equation of parabola is given by $(y-k)^2 = 4a(x-h)$ ...... (i) Where h, k are parameters Diff. w.r.t x $\Rightarrow \frac{2(y-k)dy}{dx} = 4a(1-0)$ $\Rightarrow \frac{(y-k)dy}{dx} = 2a$ ...... (ii) Diff. w.r.t. x $\Rightarrow (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ ...... (iii) From eq. (ii) $(y-k) = \frac{2a}{\frac{dy}{dx}}$ put in eq. (iii) $\Rightarrow \left(\frac{2a}{\frac{dy}{dx}}\right)\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$ $\Rightarrow 2a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ ans. 
Q.2)	Show that the D.E. representing one parameter family of curves $(x^2-y^2) = c\,(x^2+y^2)^2$ is $(x^3-3xy^2)dx = (y^3-3x^2y)dy$.
Sol.2)	We have, $(x^2-y^2) = c(x^2+y^2)^2$ ...... (i) Diff. w.r.t. x, $\Rightarrow 2x - 2y\frac{dy}{dx} = 2c(x^2+y^2).\left(2x+2y\frac{dy}{dx}\right)$ $\Rightarrow x - y\frac{dy}{dx} = 2c(x^2+y^2).\left(2x+2y\frac{dy}{dx}\right)$ Put value of $c = \frac{x^2-y^2}{(x^2+y^2)^2}$ from (i) in equation (ii) $\Rightarrow x - y\frac{dy}{dx} = 2\frac{(x^2-y^2)}{(x^2+y^2)}.(x^2+y^2)\left(x+y\frac{dy}{dx}\right)$ $\Rightarrow (x^2+y^2)\left(x-y\frac{dy}{dx}\right) = 2(x^2-y^2)\left(x+y\frac{dy}{dx}\right)$ $\Rightarrow x^3 - x^2y\frac{dy}{dx} + y^2x - y^3\frac{dy}{dx} = 2x^3 + 2x^2y\frac{dy}{dx} - 2y^2x - 2y^3\frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx}(-x^2y - y^3 - 2x^2y + 2y^3) = 2x^3 - 2y^2x - x^3 - y^2x)$ $\Rightarrow \frac{dy}{dx}(y^3 - 3x^2y) = x^3 - 3y^2x$ $\Rightarrow (y^3-3x^2y)dy = (x^3-3y^2x)dx$ (proved)
Q.3)	Find the D.E. of all non-vertical lines in a plane.
Sol.3)	Let equation of line is given by

	$\Rightarrow \frac{x}{a}+\frac{y}{b}=1$ .... (i) where a & b are parameters Diff. w.r.t. x $\Rightarrow \frac{1}{a}+\frac{1}{b}\frac{dy}{dx}=0$ ...... (ii) Diff. again w.r.t. x $\Rightarrow 0+\frac{1}{b}\frac{d^2y}{dx^2}=0$ $\Rightarrow \frac{d^2y}{dx^2}=0$ ans.
Q.4)	Show that $xy=ae^x+be^{-x}+x^2$ is a solution of the D.E. $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$
Sol.4)	We have, $xy=ae^x+be^{-x}+x^2$ ...... (i) Diff. w.r.t. x $\Rightarrow x\frac{dy}{dx}+y=ae^x-be^{-x}+2x$ Diff. again w.r.t x $\Rightarrow x\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)=ae^x+be^{-x}+2$ $\Rightarrow x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=xy-x^2+2$ $\{from\ eq.(i)ae^x+2e^{-x}=xy-x^2\}$ $\Rightarrow x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$ ∴ the given function is a solution of the given D.E. ans.
Q.5)	Verify that the function $y=c_1e^{ax}\cos(bx)+c_2e^{ax}\sin(bx)$; c_1 & c_2 are arbitrary constants is a, solution of the D.E. $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+(a^2+b^2)y=0$.
Sol.5)	We have, $y=c_1e^{ax}\cos(bx)+c_2e^{ax}\sin(bx)$ $\Rightarrow y=e^{ax}.(c_1\cos(bx))+c_2\sin(bx))$ .... (i) Diff. w.r.t. x $\Rightarrow \frac{dy}{dx}=e^{ax}(-bc_1\sin(bx)+bc_2\cos(bx))+(c_1\cos(bx)+c_2\sin(bx)).e^{ax}.a$ $\Rightarrow \frac{dy}{dx}=e^{ax}.(-bc_1\sin(bx)+bc_2\cos(bx))+ay$ .... $\{from\ eq.(i)\}$ .... (ii) Diff. again w.r.t. x $\Rightarrow \frac{d^2y}{dx^2}=e^{ax}\left(-b^2c_1\cos(bx)-b^2c_2\sin(bx)\right)+(-bc_1\sin(bx)+bc_2\cos(bx)).e^{ax}.a+a\frac{dy}{dx}\right)$ $\Rightarrow \frac{d^2y}{dx^2}=-b^2e^{ax}(c_1\cos(bx)+c_2\sin(bx))+\left(\frac{dy}{dx}-ay\right)a+a\frac{dy}{dx}$ ..... $\{from\ eq.(ii)\}$ $\Rightarrow \frac{d^2y}{dx^2}=-b^2y+a\frac{dy}{dx}-a^2y+a\frac{dy}{dx}$ $\Rightarrow \frac{d^2y}{dx^2}-2a\frac{dy}{dx}+(a^2+b^2)y=0$ Hence the given function is the solution of the given differential equation. ans.
Q.6)	Show that $y=cx+\frac{a}{c}$ is a solution of the D.E. $y=x\frac{dy}{dx}+\frac{a}{\frac{dy}{dx}}$
Sol.6)	We have, $y=cx+\frac{a}{c}$ ...... (i)

	Diff. $\frac{dy}{dx} = c$ Taking RHS $x\frac{dy}{dx} + \frac{a}{\frac{dy}{dx}}$ $= xc + \frac{a}{c}$ $= y$ ... (from eq. (i)) $=$ LHS $\therefore$ given function is a solution of the given D.E.
Q.7)	Show that the function defined by $y = \sin x - \cos x, x \in R$ is a solution of the initial value problem $\frac{dy}{dx} = \sin x + \cos x\,; y(0) = -1.$
Sol.7)	We have, $y = \sin x - \cos x$ Diff.w.r.t. x $\frac{dy}{dx} = \cos x + \sin x$, which is the given D.E. Thus, $y = \sin x - \cos x$ satisfies the D.E., hence it is a solution. Also, when $x = 0; y = \sin 0 - \cos 0 = -1$ i.e.,$y(0) = -1$ Hence, $y = \sin x - \cos x$ is a solution of the given initial value problem.