

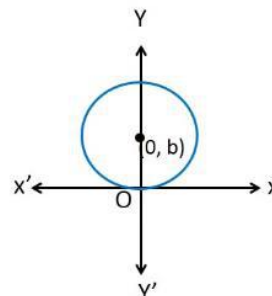
Class 12 Linear Differential Equation

Class 12th

Formation of Differential Equations

Q.1)	From the D.E. of the family of curves represented by $c(y + c)^2 = x^3$ where c is a parameter.
Sol.1)	<p>Given $c(y + c)^2 = x^3$ (i)</p> <p>Diff. wrt x, $\frac{2c(y+c)dy}{dx} = 3x^2$ (ii)</p> <p>Divide (i) \div (ii)</p> $\Rightarrow \frac{c(y+c)^2}{2c(y+c)\frac{dy}{dx}} = \frac{x^3}{3x^2}$ $\Rightarrow \frac{(y+c)}{2c(y+c)\frac{dy}{dx}} = \frac{x^3}{3x^2}$ <p>Put $y + c$ and c in eq. (i)</p> $\therefore \left(\frac{2x}{3} \frac{dy}{dx} - y\right) \left(\frac{2x}{3} \frac{dy}{dx}\right)^2 = x^3$ $\Rightarrow \left(\frac{2x}{3} \frac{dy}{dx} - y\right) \left(\frac{4x^2}{9}\right) \left(\frac{dy}{dx}\right)^2 = x^3$ $\Rightarrow \frac{8}{27} x \left(\frac{dy}{dx}\right)^3 - \frac{4y}{9} y \left(\frac{dy}{dx}\right)^2 = x$ $\Rightarrow 8x \left(\frac{dy}{dx}\right)^3 - 12y \left(\frac{dy}{dx}\right)^2 = 27x \text{ is the required solution.} \quad \text{ans.}$
Q.2)	Form the D.E. corresponding to $y^2 = a(b - x)(b + x)$ where a & b are parameters.
Sol.2)	<p>We have, $y^2 = a(b^2 - x^2)$ (i)</p> <p>Diff. w.r.t. x, $2y \frac{dy}{dx} = a(0 - 2x)$</p> $\Rightarrow y \frac{dy}{dx} = -0ax \quad \dots(ii)$ <p>Diff. again w.r.t. x, $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -a$</p> $\Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x} \frac{dy}{dx} \quad \{ \text{elementary} - a \text{ form of eq. (ii)} \}$ $\Rightarrow xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^2 - y \frac{dy}{dx} = 0 \quad \text{ans.}$
Q.3)	Form the D.E. of the family of curve $y = ae^{bx}$ where a & b are parameter.
Sol.3)	<p>We have, $y = ae^{bx}$ (i)</p> <p>Diff. wrt x</p> $\frac{dy}{dx} = bae^{bx}$ $\Rightarrow \frac{dy}{dx} = by \quad \dots(ii) \quad \dots \{ \text{from eq. (i)} y = y = ae^{bx} \}$ <p>Diff. again w.r.t. x,</p> $\frac{d^2y}{dx^2} = b \frac{dy}{dx} \quad \dots (iii)$

	$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{1}{y} \frac{dy}{dx}\right) \frac{dy}{dx} \dots \dots \left\{ \text{elementary } b = \frac{1}{y} \frac{dy}{dx} \text{ from (ii)} \right\}$ $\Rightarrow y \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 = 0 \text{ is the eq. D.E.} \quad \text{ans.}$
Q.4)	For the D.E. of the family of curve $y = Ae^{2x} + Be^{-3x}$.
Sol.4)	<p>We have, $y = Ae^{2x} + Be^{-3x} \dots (i)$</p> <p>Diff. w.r.t x,</p> $\frac{dy}{dx} = 2Ae^{2x} - 3Be^{-3x}$ $\Rightarrow \frac{dy}{dx} = 2Ae^{2x} - 3(y - Ae^{2x}) \dots \dots \{ \text{from eq. (i)} \}$ $\Rightarrow \frac{dy}{dx} = 5Ae^{2x} - 3y \dots \dots (ii)$ <p>Diff. wrt x,</p> $\frac{d^2y}{dx^2} = 10Ae^{2x} - 3 \frac{dy}{dx} \dots (iii)$ <p>From (ii) $Ae^{2x} = \frac{1}{5} \left(\frac{dy}{dx} + 3y \right)$ put in eq. (iii)</p> $\frac{d^2y}{dx^2} = 10 \left[\frac{1}{5} \left(\frac{dy}{dx} + 3y \right) \right] - 3 \frac{dy}{dx}$ $\Rightarrow \frac{d^2y}{dx^2} = 2 \frac{dy}{dx} + 6y - 3 \frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} = 6y \quad \text{ans.}$
Q.5)	Form the D.E. of the family of curve $y = e^x (A \cos x + B \sin x)$.
Sol.5)	<p>We have, $y = e^x (A \cos x + B \sin x) \dots (i)$</p> <p>Diff. w.r.t. x,</p> $\frac{dy}{dx} = e^x (-A \sin x + B \cos x) + (A \cos x + B \sin x) e^x$ $\Rightarrow \frac{dy}{dx} = e^x (-A \sin x + B \cos x) + y \dots (ii) \quad (\text{from eq. (i)})$ <p>Diff. again w.r.t. x,</p> $\frac{d^2y}{dx^2} = e^x (-A \cos x - B \sin x) + (-A \sin x + B \cos x) e^x + \frac{dy}{dx}$ $\Rightarrow \frac{d^2y}{dx^2} = -y + \frac{dy}{dx} - y + \frac{dy}{dx} \dots (using (i) \& (ii))$ $\Rightarrow \frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0 \text{ is the required D.E.} \quad \text{ans.}$
Q.6)	Find the D.E. of all the circle touching x - axis at the origin.
Sol.6)	<p>Equation of circle touching x - axis at the origin is given by</p> $(x - 0)^2 + (y - k)^2 = k^2 \dots (i)$ <p>Where k is a parameter</p> <p>Diff. w.r.t x,</p> $2x + 2(y - k) \frac{dy}{dx} = 0$ $\Rightarrow y - k = \frac{-x}{\frac{dy}{dx}}$ $\Rightarrow k = y + \frac{x}{\frac{dy}{dx}}$ <p>Put $y - k$ and k in eq. (i)</p>



	$x^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} = \left(y + \frac{x}{\frac{dy}{dx}}\right)^2$ $\Rightarrow x^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} = y^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} + \frac{2xy}{\frac{dy}{dx}}$ $\Rightarrow \frac{(x^2 - y^2)dy}{dx} = 2xy \text{ is the required D.E.} \quad \text{ans.}$
Q.7)	Form the differential equation of family of circles in the second quadrant and touching the coordinate axis.
Sol.7)	<p>The equation is of the form $(x + a)^2 + (y - a)^2 = a^2$</p> <p>Differentiating w.r.t x</p> $2(x + a) + 2(y - a) \cdot y' = 0$ <p>Therefore $a = \frac{x + yy'}{y' - 1}$</p> <p>Substituting in the first equation,</p> $\left(x + \frac{x + yy'}{y' - 1}\right)^2 + \left(y - \frac{x + yy'}{y' - 1}\right)^2 = \left[\frac{x + yy'}{y' - 1}\right]^2$ <p>On expanding and simplifying we get,</p> $(xy' + yx')^2 + (2yy' + x - y)^2 = (x + yy')^2 \text{ is the required solution} \quad \text{ans.}$
Q.8)	Obtain the D.E. of all the circles with radius r .
Sol.8)	<p>Let centre is (a, b) the equation of circle is</p> $(x - a)^2 + (y - b)^2 = r^2 \quad \dots (i)$ <p>Diff. w.r.t. x,</p> $2(x - a) + 2(y - b) \frac{dy}{dx} = 0$ $\Rightarrow x - a + (y - b) \frac{dy}{dx} = 0 \quad \dots (ii)$ <p>Diff. again w.r.t x,</p> $\Rightarrow 1 + (y - b) \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ $\Rightarrow (x - a) = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)}{\frac{d^2y}{dx^2}} \cdot \frac{dy}{dx} = 0$ $\Rightarrow x - a = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right] \cdot \frac{dy}{dx}}{\left(\frac{d^2y}{dx^2}\right)}$ <p>Put value of $(x - a)$ & $(y - b)$ in eq. (i) we get</p> $\frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right) \cdot \left(\frac{dy}{dx}\right)}{\left(\frac{d^2y}{dx^2}\right)^2} + \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$ $\Rightarrow \left(1 + \left(\frac{dy}{dx}\right)^2\right)^2 \left(\left(\frac{dy}{dx}\right)^2 + 1\right) = r^2 \left(\frac{d^2y}{dx^2}\right)^2 = r^2 \left(\frac{d^2y}{dx^2}\right)^2$ $\Rightarrow \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = r^2 \cdot \left(\frac{d^2y}{dx^2}\right) \text{ is the required D.E.} \quad \text{ans.}$

Q.9)	Find the D.E. of the family of parabolas having their axis of symmetry coincident with the axis of x .
Sol.9)	<p>Vertex is (h, a)</p> <p>Equation of parabola is given by</p> $(y - 0)^2 = 4a(x - h) \dots\dots\dots (i)$ <p>Where a & h are parameter</p> <p>Diff. w.r.t. x,</p> $2y \frac{dy}{dx} = 4a(1 - 0)$ $\Rightarrow y \frac{dy}{dx} = 2a \dots\dots (ii)$ <p>Diff. again w.r.t. x,</p> $y \left(\frac{d^2y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2 = 0 \text{ is the required D.E.} \quad \text{ans.}$
Q.10)	Form the D.E. of the family of ellipses having foci on y - axis and centre at the origin.
Sol.10)	<p>Equation of ellipse is</p> $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$ <p>Where a & b are parameter</p> <p>Diff. w.r.t. x,</p> $\frac{2x}{a^2} + \frac{2y \left(\frac{dy}{dx} \right)}{b^2} = 0$ $\Rightarrow \frac{x}{a^2} + \frac{y \frac{dy}{dx}}{b^2} = 0 \dots\dots (ii)$ <p>Diff. again w.r.t x,</p> $\frac{1}{a^2} + \frac{y \left(\frac{d^2y}{dx^2} \right) + \left(\frac{dy}{dx} \right)^2}{b^2} = 0 \dots\dots (iii)$ <p>From eq. (ii) $\frac{1}{a^2} = -\frac{y \frac{dy}{dx}}{b^2 x}$ put in eq.(iii)</p> $\Rightarrow -\frac{y \frac{dy}{dx}}{b^2 x} + -\frac{y \left(\frac{dy}{dx} \right)^2 + \left(\frac{dy}{dx} \right)^2}{b^2} = 0$ $\Rightarrow -y \frac{dy}{dx} x \cdot y \left(\frac{dy}{dx} \right)^2 + x \left(\frac{dy}{dx} \right)^2 = 0 \quad \text{ans.}$

