Class 12 Linear Differential Equation

Class 12th

Q.1)	Solve the DE $(x+y)\frac{dy}{dx}=1$.
Sol.1)	$\frac{dy}{dx}=\frac{1}{x+y}$ $\Rightarrow \frac{dx}{dy}=x+y$ $\Rightarrow \frac{dx}{dy}-x=y$ Comparing with $\frac{dx}{dy}+Px=\theta$ Here $P=-1$ & $\theta=y$ $I.F.=e^{-\int 1dy}=e^{-y}$ Solution is given by $x.(I.F.)=\int \theta\,(I.F.)dy+c$ $\Rightarrow x.e^{-y}=\int y.e^{-y}+c$ $\Rightarrow x.e^{-y}=y\frac{e^{-y}}{-1}-\int 1.\frac{e^{-y}}{-1}dy+c$ $\Rightarrow x.e^{-y}=-ye^{-y}+\int e^{-y}dy+c$ $\Rightarrow x.e^{-y}=-ye^{-y}-e^{-y}+c$ $\Rightarrow xe^{-y}=-e^{-y}(y+1)+c$ $\Rightarrow x=-(y+1)+ce^{y}$ ans.
Q.2)	Solve the D.E. $x\frac{dy}{dx}-y=(x+1)e^{-x}\,;y(1)=0$
Sol.2)	Divide by x $\frac{dy}{dx}-\frac{y}{x}=\frac{x+1}{x}e^{-x}$ Comparing with $\frac{dy}{dx}+Py=\theta$ Here $P=-\frac{1}{x}$ and $\theta=\frac{x+1}{x}.e^{-x}$ $I.F.=e^{\int\frac{1}{x}dx}=e^{-\log x}=e^{\log x^{-1}}=\frac{1}{x}$ Solution is given by $y.(I.F.)=\int \theta\,.(I.F.)dx+c$ $\Rightarrow y.\frac{1}{x}=\int \frac{x+1}{x}e^{-x}.\frac{1}{x}dx+c$ $\Rightarrow \frac{y}{x}=\int\left(\frac{1}{x}+\frac{1}{x^2}\right)e^{-x}dx+c$ $\Rightarrow \frac{y}{x}=\int e^{-x}.\frac{1}{x}dx+\int e^{-x}.\frac{1}{x^2}dx+c$ $\Rightarrow \frac{y}{x}=\frac{1}{x}.\frac{e^{-x}}{(-1)}-\frac{1}{x^2}.\frac{e^{-x}}{(-1)}dx+\int e^{-x}.\frac{1}{x^2}dx+c$ $\Rightarrow \frac{y}{x}=-\frac{1}{x}e^{-x}-\int\frac{1}{x^2}e^{-x}dx+\int\frac{1}{x^2}e^{-x}dx+c$ $\Rightarrow \frac{y}{x}=-\frac{1}{x}e^{-x}+c$

	Put $x = 1$ and $y = 0$ $\Rightarrow 0 = -e^{-1} + c \Rightarrow c = \frac{1}{e}$ $\therefore \frac{y}{x} = -\frac{1}{x}e^{-x} + \frac{1}{e}$ $\Rightarrow y = -e^{-x} + xe^{-1}$ is the required solution ans.
Q.3)	Show that the D.E. is homogeneous D.E. & also find the particular solution. $(x+y)dy + (x-y)dx = 0; y = 1$ when $x = 1$
Sol.3)	We have , $(x+y)dy + (x-y)dx = 0$ $\Rightarrow \frac{dy}{dx} = \frac{-(x-y)}{x+y} = \frac{y-x}{x+y}$ .... (i) Here $f(x,y) = \frac{y-x}{x+y}$ $f(\lambda x.\lambda y) = \frac{\lambda y-\lambda x}{\lambda x+\lambda y}$ $f(\lambda x,\lambda y) = \frac{\lambda(y-x)}{\lambda(x+y)}$ $f(\lambda x,\lambda y) = \lambda^0 f(x.,y)$ Clearly function is homogeneous function of degree 0 $\therefore$ D.E. is a homogeneous D.E. Put $y = vx$ Diff. w.r.t. $x, \frac{dy}{dx} = v + \frac{xdv}{dx}$ $\therefore$ equation (i) becomes $v + \frac{xdv}{dx} = \frac{vx-x}{x+vx}$ $\Rightarrow v + \frac{xdv}{dx} = \frac{v-1}{1+v}$ $\Rightarrow \frac{xdv}{dx} = \frac{v-1}{1+v} - v$ $\Rightarrow \frac{xdv}{dx} = \frac{v-1-v-v^2}{1+v}$ $\Rightarrow \frac{xdv}{dx} = \frac{-(v^2+1)}{1+v}$ $\Rightarrow \frac{1+v}{v^2+1}dv = -\frac{dx}{x}$........ (separately variables) Interpreting both sides $\Rightarrow \int \frac{1+v}{1+v^2}dv = -\int \frac{dx}{x}$ Separate : $\Rightarrow \int \frac{1}{1+v^2}dv + \int \frac{v}{1+v^2}dv = -\log\lvert x\rvert$ Put $1 + v^2 = t\,; vdv = \frac{dt}{2}$ $\Rightarrow \tan^{-1} v + \frac{1}{2}\int \frac{dt}{t} = -\log\lvert x\rvert$ $\Rightarrow \tan^{-1} v + \frac{1}{2}\log\lvert v^2+1\rvert = -\log\lvert x\rvert + c$ $\Rightarrow 2\tan^{-1} v + \log\lvert v^2+1\rvert = -2\log\lvert x\rvert + 2c$ $\Rightarrow 2\tan^{-1} v + \log\lvert v^2+1\rvert + \log\lvert x\rvert^2 = 2c$

	Replace v by $\frac{y}{x}$ $\Rightarrow 2\tan^{-1}\left(\frac{y}{x}\right)+\log\left\|\left(\frac{y^2}{x^2}+1\right).x^2\right\|=2c$ $\Rightarrow 2\tan^{-1}\left(\frac{y}{x}\right)+\log\|x^2+y^2\|=2c$ Put $x=1$ and $y=1$ $\Rightarrow 2\tan^{-1}(1)+\log(2)=2c$ $\Rightarrow 2\left(\frac{\pi}{4}\right)+\log(2)=2c$ $\Rightarrow 2c=\frac{\pi}{2}+\log 2$ Solution is given by $\therefore 2\tan^{-1}\left(\frac{y}{x}\right)+\log\|x^2+y^2\|=\frac{\pi}{2}+\log 2$ ans.
Q.4)	Show that D.E. is homogeneous & also find the initial value problem $(x^2+xy)dy=(x^2+y^2)dx$ given $y(1)=0$
Sol.4)	We have, $\frac{dy}{dx}=\frac{x^2+y^2}{x^2+xy}$ .....(i) Here $f(x,y)=\frac{x^2+y^2}{x^2+xy}$ $f(\lambda x.\lambda y)=\frac{\lambda^2x^2+\lambda^2y^2}{\lambda^2x^2+\lambda^2xy}$ $ff(\lambda x,\lambda y)=\lambda^0.f(x.,y)$ Clearly function is homogeneous of degree 0 $\therefore$ D.E. is homogeneous D.E. Now put $y=vx$ Diff. w.r.t x, $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore$ equation (i) becomes $v+\frac{xdv}{dx}=\frac{x^2+v^2x^2}{x^2+vx^2}$ $\Rightarrow v+\frac{xdv}{dx}=\frac{1+v^2}{1+v}$ $\Rightarrow \frac{xdv}{dx}=\frac{1+v^2}{1+v}-v$ $\Rightarrow \frac{xdv}{dx}=\frac{1+v^2-v-v^2}{1+v}$ $\Rightarrow \frac{xdv}{dx}=\frac{1-v}{1+v}$ $\Rightarrow \frac{xdv}{dx}=\frac{-(v-1)}{v+1}$ $\Rightarrow \frac{v+1}{v-1}dv=\frac{-dx}{x}$....... (separately variables) Interpreting both sides $\int\frac{v+1}{v-1}dx=-\int\frac{dx}{x}$ Adjustment $\int\frac{v+1-1+1}{v-1}dx=-\log\|x\|$

	$\Rightarrow \int \frac{(v-1)+2}{v-1} dv = -\log\lvert x\rvert$ $\Rightarrow \int 1 + \frac{2}{v-1} dv = -\log\lvert x\rvert$ $\Rightarrow v + 2\log\lvert v-1\rvert = -\log\lvert x\rvert + c$ Replace v by $\frac{y}{x}$ $\Rightarrow \frac{y}{x} + 2\log\left\lvert\frac{y}{x} - 1\right\rvert + \log\lvert x\rvert + c$ $\Rightarrow \frac{y}{x} + \log\left\lvert\left(\frac{y-x}{x}\right)^2 . x\right\rvert \pm c$ Put $x = 1$ & $y = 0$ $\Rightarrow 0 + \log\lvert 1\rvert = c \Rightarrow c = 0$ $\therefore \frac{y}{x} + \log\left\lvert\frac{(y-x)^2}{x}\right\rvert = 0$ $\Rightarrow \log\left\lvert\frac{(y-x)^2}{x}\right\rvert = \frac{-y}{x}$ $\Rightarrow \frac{(y-x)^2}{\lvert x\rvert} = e^{-\frac{y}{x}}$ $\Rightarrow (x-y)^2 = \lvert x\rvert\, e^{-\frac{y}{x}}$ is the required solution ans.
Q.5)	Find the general solution: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$
Sol.5)	$\frac{dy}{dx} = \frac{x^3-3xy^2}{y^3-3x^2y}$ Clearly the degree of each term in nominator & denominator is same. It is a homogeneous D.E. Put $y = vx$ Diff. w.r.t x; $\frac{dy}{dx} = v + \frac{xdv}{dx}$ $\therefore$ equation (i) becomes $v + \frac{xdv}{dx} = \frac{x^3-3v^2x^3}{v^3x^3-3vx^3}$ $\Rightarrow v + \frac{xdv}{dx} = \frac{1-3v^2}{v^3-3v}$ $\Rightarrow \frac{xdv}{dx} = \frac{1-3v^2}{v^3-3v} - v$ $\Rightarrow \frac{xdv}{dx} = \frac{-v^4-1}{v^3-3v}$ $\Rightarrow \frac{v^3-3v}{v^4-1} dv = \frac{-dx}{x}$ Interpreting both sides $\int \frac{v^3-3v}{v^4-1} dv = -\int \frac{dx}{x}$ Separate $\int \frac{v^3}{v^4-1} dv - 3\int \frac{v}{v^4-1} dv = -\log\lvert x\rvert$ Put $v^4 - 1 = t$ Put $v^2 = z$ in 2nd integral $v^3 dv = \frac{dt}{4}$ $vdv = \frac{dz}{2}$

	$\Rightarrow \frac{1}{4}\int \frac{dt}{t} - \frac{3}{2}\int \frac{dz}{z^2-1} = -\log\lvert x\rvert$ $\Rightarrow \frac{1}{4}\log\lvert t\rvert - \frac{3}{2} \times \frac{1}{2}\log\left\lvert\frac{z-1}{z+1}\right\rvert = -\log\lvert x\rvert + \log c$ $\Rightarrow \frac{1}{4}\log\lvert v^4-1\rvert - \frac{3}{4}\log\left\lvert\frac{v^2-1}{v^2+1}\right\rvert = -\log\lvert x\rvert + \log c$ $\Rightarrow \log\lvert v^4-1\rvert - 3\log\left\lvert\frac{v^2-1}{v^2+1}\right\rvert = -4\log\lvert x\rvert + 4\log c$ Replace v by $\frac{y}{x}$ $\Rightarrow \log\left\lvert\frac{y^4-x^4}{x^4}\right\rvert - 3\log\left\lvert\frac{y^2-x^2}{y^2+x^2}\right\rvert + 4\log\lvert x\rvert = \log c^4$ $\Rightarrow \log\left\lvert\frac{\left(\frac{y^4-x^4}{x^4}\right).x^4}{\left(\frac{y^2-x^2}{y^2+x^2}\right)^3}\right\rvert = \log c^4$ $\Rightarrow \log\left\lvert(y^4-x^4).\left(\frac{y^2+x^2}{y^2-x^2}\right)^3\right\rvert = \log c^4$ $\Rightarrow (y^2+x^2)(y^2-x^2)\frac{(y^2+x^2)^3}{(y^2-x^2)^3} = c^4$ $\Rightarrow \frac{(x^2+y^2)}{(x^2-y^2)^2} = c^4$ $\Rightarrow (x^2+y^2)^2 = (x^2-y^2)c^2$ is the required solution ans.
Q.6)	Find the particular solution of the D.E. $(3xy+y^2)dx + (x^2+xy)dy = 0, x = 1\ \&\ y = 1$
Sol.6)	We have, $(3xy+y^2)dx + (x^2+xy)dy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{-(3xy+y^2)}{x^2+xy}$ ………… (i) It is a homogeneous D.E. Put $y = vx$ Diff. w.r.t. $x, \frac{dy}{dx} = v + \frac{xdv}{dx}$ ∴ equation (i) becomes $v + \frac{xdv}{dx} = \frac{-(3vx^2+v^2x^2)}{x^2+vx^2}$ $\Rightarrow v + x\frac{dv}{dy} = \frac{-(3v+v^2)}{1+v}$ $\Rightarrow \frac{xdv}{dx} = \frac{-3v-v^2}{1+v} - v$ $\Rightarrow \frac{xdv}{dx} = \frac{-3v-v^2-v-v^2}{1+v}$ $\Rightarrow \frac{xdv}{dx} = \frac{-(2v^2+4v)}{v+1}$ $\Rightarrow \frac{v+1}{2v^2+4v}dv = \frac{-dx}{x}$ (separate variables) Interpreting both sides $\Rightarrow \int \frac{v+1}{2v^2+4v}dv = -\int \frac{dx}{x}$ Put $2v^2+4v = t$ $(4v+4)dv = dt$

	$\Rightarrow (v+1)dv = \frac{dt}{4}$ $\therefore \frac{1}{4}\int \frac{dt}{t} = -\int \frac{dx}{x}$ $\frac{1}{4}\log 2v^2 + 4v = -\log x + \log c$ $\Rightarrow \log 2v^2 + 4v = -4\log x + 4\log c$ Replace v by $\frac{y}{x}$ $\Rightarrow \log \frac{2y^2}{x^2} + \frac{4y}{x} = \log \frac{c^4}{x^4}$ $\Rightarrow \log \frac{2y^2+4xy}{x^2} = \log\left(\frac{c^4}{x^4}\right)$ $\Rightarrow \frac{\lvert 2y^2+4xy\rvert}{x^2} = \frac{c^4}{x^4}$ $\Rightarrow \lvert 4xy + 2y^2\rvert = \frac{c^1}{x^2}$ ans.
Q.7)	Find one parameter solution of the D.E. $x\cos\left(\frac{y}{x}\right).(ydx + x\,dy) = y\sin\left(\frac{y}{x}\right).(x\,dy - ydx)$
Sol.7)	$x\,y\cos\left(\frac{y}{x}\right)dx + x^2\cos\left(\frac{y}{x}\right)dy = xy\sin\left(\frac{y}{x}\right)dy - y^2\sin\left(\frac{y}{x}\right)$ $\Rightarrow dy\left(x^2\cos\left(\frac{y}{x}\right) - xy\sin\left(\frac{y}{x}\right)\right) = -dx\left(y^2\sin\left(\frac{y}{x}\right) + xycos\left(\frac{y}{x}\right)\right)$ $\frac{dy}{dx} = \frac{-\left(y^2\sin\left(\frac{y}{x}\right)+xy\cos\left(\frac{y}{x}\right)\right)}{x^2\cos\left(\frac{y}{x}\right)-xy\sin\left(\frac{y}{x}\right)}$ .... (i) It is a homogeneous D.E. Put $y = vx$ Diff. w.r.t. $x, \frac{dy}{dx} = v + \frac{xdv}{dy}$ put in eq. (i) $\Rightarrow v + \frac{xdv}{dx} = \frac{-v^2x^2\sin v + vx^2\cos v}{x^2\cos v - vx^2\sin v}$ $\Rightarrow v + \frac{xdv}{dx} = \frac{-(v^2\sin v + v\cos v)}{\cos v - v\sin v}$ $\Rightarrow \frac{xdv}{dx} = \frac{-v^2\sin v + v\cos v}{\cos v - v\sin v} - v$ $\Rightarrow \frac{xdv}{dx} = \frac{-v^2\sin v - v\cos v - \cos v + v^2\sin v}{\cos v - v\sin v}$ $\Rightarrow \frac{xdv}{dx} = \frac{-2v\cos v}{\cos v - v\sin v}$ $\Rightarrow \frac{\cos v - v\sin v}{v\cos v} = \frac{-2dx}{x}$ Interpreting both sides $\Rightarrow \int \frac{\cos v - v\sin v}{v\cos v}dv = -2\int \frac{dx}{x}$ $\Rightarrow \int \frac{1}{v} - \tan v\ \ dx = -\log x$ $\Rightarrow \log v - \log\sec v = -\log x^2 + \log c$ $\Rightarrow \log \frac{v}{\sec v} = -\log x^2 + \log c$ $\Rightarrow \frac{xy}{\sec\left(\frac{y}{x}\right)} = c$

	$\Rightarrow \log \frac{\left(\frac{y}{x}\right)}{\sec\left(\frac{y}{x}\right)} . x^2 = \log c$ $\Rightarrow \log \frac{xy}{\sec\left(\frac{y}{x}\right)} = \log c$ $\Rightarrow \frac{xy}{\sec\left(\frac{y}{x}\right)} = \pm c$ $\Rightarrow xy = c^1 \sec\left(\frac{y}{x}\right)$; where $c^1 = \pm c$ is the required solution ans.
Q.8)	Find the particular solution of the D.E. $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = \frac{x + y\cos\left(\frac{y}{x}\right)}{x\cos\left(\frac{y}{x}\right)}$
Sol.8)	It is a homogeneous D.E. Put $y = vx$ Diff. w.r.t $x, \frac{dy}{dx} = v + \frac{xdv}{dx}$ $\therefore v + \frac{xdv}{dx} = \frac{x + vx\cos v}{x\cos v}$ $\Rightarrow v + \frac{xdv}{dx} = \frac{1 + v\cos v}{\cos v}$ $\Rightarrow \frac{xdv}{dx} = \frac{1 + v\cos v}{\cos v} - v$ $\Rightarrow \frac{xdv}{dx} = \frac{1 + v\cos v - v\cos v}{\cos v}$ $\Rightarrow \frac{xdv}{dx} = \frac{1}{\cos v}$ $\Rightarrow \cos v \ dv = \frac{dx}{x}$ Interpreting both sides $\int \cos v \ dv = \frac{dx}{x}$ $\Rightarrow \sin v = \log\lvert x\rvert + c$ Replace v by $\frac{y}{x}$ $\therefore \sin\left(\frac{y}{x}\right) = \log\lvert x\rvert + c$ Put $x = 1$ and $y = \frac{r}{4}$ $\Rightarrow \sin\left(\frac{r}{4}\right) = \log\lvert 1\rvert + c$ $\Rightarrow \frac{1}{\sqrt{2}} = c$ $\therefore \sin\left(\frac{y}{x}\right) = \log\lvert x\rvert + \frac{1}{\sqrt{2}}$ ans.
Q.9)	Show that D.E. is homogeneous & solve it $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$
Sol.9)	We have, $2ye^{\frac{x}{y}}dx = -\left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ $\Rightarrow \frac{dx}{dy} = \frac{2ye^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}$ ………… (i)

Here $f(x,y)=\frac{2ye^{\frac{x}{y}}-y}{2ye^{\frac{x}{y}}}$

$f(\lambda x,\lambda y)=\frac{2\lambda xe^{\frac{x}{y}}-\lambda y}{2\lambda ye^{\frac{x}{y}}}$

$=\frac{\lambda}{\lambda}\left(\frac{2xe^{\frac{x}{y}}-y}{2ye^{\frac{x}{y}}}\right)$

$f(\lambda x,\lambda y)=\lambda^0 f(x,y)$

Clearly function is homogeneous D.E.

Put $x=vy$

Diff. w.r.t. $y, \frac{dx}{dy}=v+y\frac{dv}{dy}$

$\therefore$ equation (i) become

$v+y\frac{dv}{dy}=\frac{2vye^v-y}{2ye^v}$

$\Rightarrow v+y\frac{dv}{dy}=\frac{2vye^v-1}{2e^v}$

$\Rightarrow y\frac{dv}{dy}=\frac{2e^v.v-1}{2e^v}-v$

$\Rightarrow y\frac{dv}{dy}=\frac{2ve^v-1-2v^e}{2e^v}$

$\Rightarrow y\frac{dv}{dy}=-\frac{1}{2e^v}$

$\Rightarrow e^v dv=-\frac{1}{2}\frac{dy}{y}$

Interpreting both sides

$\int e^v dv=-\frac{1}{2}\int\frac{dy}{y}$

$\Rightarrow e^v=-\frac{1}{2}\log|y|+c$

Replace v by $\frac{x}{y}$

$\Rightarrow e^{\frac{x}{y}}=-\frac{1}{2}\log|y|+c$ is the required solution ans.

Q.10) Solve the D.E.

$xe^{\frac{y}{x}}-y\sin\left(\frac{y}{x}\right)+x\frac{dy}{dx}.\sin\left(\frac{y}{x}\right)=0; y(1)=0$

Sol.10) We have, $xe^{\frac{y}{x}}-y\sin\left(\frac{y}{x}\right)+x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)=0$

$\Rightarrow\frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-xe^{\frac{y}{x}}}{x\sin\left(\frac{y}{x}\right)}$ .... (i)

It is homogeneous D.E.

Put $y=vx$

Diff. w.r.t $x, \frac{dy}{dx}=v+x\frac{dv}{dx}$ Put in eq. (i)

$\therefore v+\frac{xdv}{dx}=\frac{vx\sin v-xe^v}{x\sin v}$

	$\Rightarrow v + \frac{xdv}{dx} = \frac{v\sin v - e^v}{\sin v}$ $\Rightarrow \frac{xdv}{dx} = \frac{v\sin v - e^v}{\sin v} - v$ $\Rightarrow \frac{xdv}{dx} = \frac{v\sin v - e^v - v\sin v}{\sin v}$ $\Rightarrow \frac{xdv}{dx} = \frac{-e^v}{\sin v}$ $\Rightarrow \frac{\sin v}{e^v} dv = \frac{-dx}{x}$ $\Rightarrow \int e^{-v} \sin v \ dv = -\int \frac{dx}{x}$ ......(ii) Let $I = \int e^{-v} \sin v\, dv$ $I = sin\, v.\frac{e^{-v}}{-1} - \int \cos v\,.\frac{e^{-v}}{-1} dv$ $I = -e^{-v}\sin v + \int e^{-v}\cos v\, dv$ $I = -e^{-v}\sin v + \cos v.\frac{e^{-v}}{-1} - \sin v.\frac{e^{-v}}{-1} dv$ $I = -e^{-v}\sin v + \cos v.\frac{e^{-v}}{-1} - \sin v.\frac{e^{-v}}{-1} dv$ $I = -e^{-v}\sin v - \cos v.e^{-v} - I$ $2I = -e^{-v}(\sin v + \cos v)$ $I = -\frac{e^{-v}}{2}(\sin v + \cos v)$ ∴ equation (ii) becomes $\frac{-e^{-v}}{2}(\sin v + \cos v) = -\log\lvert x\rvert + c$ $\Rightarrow e^{-v}(\sin v + \cos v) = 2\log\lvert x\rvert - 2c$ Replace v by $\frac{y}{x}$ $\Rightarrow e^{-\frac{y}{x}}\left(\sin\frac{y}{x} + \cos\frac{y}{x}\right) = \log\lvert x^2\rvert - 2c$ Put $x = 1$ and $y = 0$ $\Rightarrow e^0(\sin + \cos 0) = \log\lvert 1\rvert - 2c$ $\Rightarrow 1 = -2c \Rightarrow c = -\frac{1}{2}$ $\therefore e^{-\frac{y}{x}}\left(\sin\frac{y}{x} + \cos\frac{y}{x}\right) = \log\lvert x^2\rvert + 1$ is the required particular solution ans.