Class XII

Class 12 Linear Differential Equation

Q.1)	Solve the D.E : $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x.$
Sol.1)	Divide by $x\log x$ $\frac{dy}{dx}+\frac{1}{x\log x}y=\frac{2}{x^2}$ Comparing with $\frac{dy}{dx}+Py=\theta$ We have, $P=\frac{1}{x\log x}$ & $\theta=\frac{2}{x^2}$ $I.F.=e^{\int Pdx}=e^{\int\frac{1}{x\log x}dx}$ Put $\log x=t\Rightarrow\frac{1}{x}dx=dt$ $\therefore I.F=e^{\int\frac{dt}{t}}=e^{\int\log t}=t=\log x$ $\therefore I.F.=\log x$ Solution is given by $y.(I.F.)=\int\theta.(I.F.)dx+C$ $\Rightarrow y.\log x=2\int\frac{1}{x^2}.\log x\,dx+C$ $\Rightarrow y\log x=2\left[\log x\left(\frac{-1}{x}\right)-\int\frac{1}{x}\left(\frac{-1}{x}\right)dx\right]+C$ $\Rightarrow y\log x=2\left[\frac{-\log x}{x}+\int\frac{1}{x^2}dx\right]+C$ $\Rightarrow y\log x=2\left[\frac{-\log x}{x}-\frac{1}{x}\right]+C$ $\Rightarrow y\log x=\frac{-2}{x}(\log x+1)+C$ is the required solution.
Q.2)	Solve the D.E. : $x\frac{dy}{dx}+y-x+xy\cot x=0.$
Sol.2)	We have, $x\frac{dy}{dx}+y(1+x\cot x)=x$ Divide by x $\Rightarrow\frac{dy}{dx}+y\left(\frac{1}{x}+\cot x\right)=1$ Comparing with $\frac{dy}{dx}+Py=\theta$ We have, $P=\frac{1}{x}+\cot x$ and $\theta=1$ $I.F.=e^{\int\frac{1}{x}+\cot x dx}=e^{\log x\log(\sin x)}$ $=e^{\log(x\sin x)}=x\sin x$ $\therefore I.F.=x\sin x$ Solution is given by $\Rightarrow y(I.F.)=\int\theta(I.F.)dx+C$ $\Rightarrow y\,(x\sin x)=\int 1.(x\sin x)dx+C$ $\Rightarrow y(x\sin x)=x(-\cos x)-\int(1).(-\cos x)dx+C$

	$\Rightarrow y(x\sin x) = -x\cos x + \sin x + C$ ans.
Q.3)	Solve the initial value problem $(x^2+1)y' - 2xy = (x^4+2x^2+1)\cos x\,; y(0) = 0.$
Sol.3)	We have, $(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)^2.\cos x$ Divide by x^2+1 $\Rightarrow \frac{dy}{dx} - \frac{2x}{1+x^2}y = (x^2+1).\cos x$ Comparing with $\frac{dy}{dx} + Py = \theta$ We have, $P = \frac{-2x}{x^2+1}$ and $\theta = (x^2+1)\cos x$ $I.F. = e^{\int Pdx} = e^{-\int\frac{2x}{1+x^2}dx}$ Put $1+x^2 = t$ $\Rightarrow 2xdx = dt$ $\therefore I.F. = e^{-\int\frac{dt}{t}} = e^{-\log t} = e^{\log(t)^{-1}}$ $I.F. = \frac{1}{t} = \frac{1}{1+x^2}$ $\therefore I.F. = \frac{1}{1+x^2}$ Now solution is given by $y(I.F.) = \int \theta.(I.F.)dx + C$ $\Rightarrow y \times \frac{1}{x^2+1} = \int(x^2+1).\cos x.\frac{1}{1+x^2}dx + C$ $\Rightarrow \frac{y}{x^2+1} = \sin x + C$ Initial condition $y(0) = 0 \Rightarrow x = 0$ and $y = 0$ $\Rightarrow 0 = \sin(0) = C$ $\Rightarrow C = 0$ $\therefore \frac{y}{x^2+1} = \sin x$ $\Rightarrow y = (x^2+1)\sin x$ is the required particular solution.
Q.4)	Solve the D.E. $\frac{dy}{dx} = \frac{-x+y\cos x}{1+\sin x}$.
Sol.4)	We have, $\frac{dy}{dx} = \frac{-x+y\cos x}{1+\sin x}$ $\Rightarrow \frac{dy}{dx} = \frac{-x}{1+\sin x} - \frac{y\cos x}{1+\sin x}$ $\Rightarrow \frac{dy}{dx} + \frac{y\cos x}{1+\sin x} = \frac{-x}{1+\sin x}$ Comparing with $\frac{dy}{dx} + Py = \theta$ We have, $P = \frac{\cos x}{1+\sin x}$ and $\theta = \frac{-xy}{1+\sin x}$ $I.F. = e^{\int\frac{\cos x}{1+\sin x}}$ Put $1+\sin x = t \Rightarrow \cos x\,dx = dt$ $\therefore I.F. = e^{\int\frac{dt}{t}} = e^{\log t} = t = 1+\sin x$ $\therefore I.F. = 1+\sin x$ Solution is given by $y(I.F.) = \int \theta.(I.F.)dx + C$

	$\Rightarrow y(1+\sin x) = \int \frac{-x}{(1+\sin x)} \times (1+\sin x)dx + C$ $\Rightarrow y(1+\sin x) = \frac{(-x)^2}{2} + C$ is the required solution.
Q.5)	Find the general solution $\frac{dy}{dx} - 2y = \cos(3x)$
Sol.5)	Comparing with $\frac{dy}{dx} + Py = \theta$ We have, $P = -2$ and $\theta = \cos(3x)$ $I.F. = e^{\int -2dx}$ $I.F. = e^{-2x}$ Solution is given by $y(I.F.) = \int \theta\,(I.F.)dx + C$ $y.e^{-2x} = \int e^{-2x}\cos(3x)\,dx + C$ $\Rightarrow y.e^{-2x} = I + C$ .... (i) Where $I = \int e^{-2x}\cos(3x)\,dx$ $I = \cos(3x).\frac{e^{-2x}}{-2} - \int -3\sin(3x).\frac{e^{-2x}}{-2}dx$ $I = \frac{-1}{2}\cos(3x).e^{-2x} - \frac{3}{2}\int e^{-2x}.\sin(3x)\,dx$ $= -\frac{1}{2}.\cos(3x)\,e^{-2x} - \frac{3}{2}\left[\sin(3x).\frac{e^{-2x}}{-2} - \int 3\cos(3x).\frac{e^{-2x}}{-2}dx\right]$ $= -\frac{1}{2}\cos(3x)\,e^{-2x} - \frac{3}{2}\left[-\frac{1}{2}\sin(3x)\,e^{-2x} + \frac{3}{2}\,I\right]$ $I = -\frac{1}{2}.e^{-2x}.\cos(3x) + \frac{3}{4}e^{-2x}.\sin(3x) - \frac{9}{4}I$ $\Rightarrow I + \frac{9}{4}I = \frac{e^{-2x}}{4}(-2\cos(3x) + 3\sin(3x))$ $\Rightarrow 13I = e^{-2x}(-2\cos(3x) + 3\sin(3x))$ $\Rightarrow I = \frac{e^{-2x}}{13}(-2\cos(3x) + 3\sin(3x))$ $\therefore$ equation (i) become $y.e^{-2x} = \frac{e^{-2x}}{13}(-2\cos(3x) + 3\sin(3x)) + C$ $\Rightarrow y = \frac{1}{13}(-2\cos(3x) + 3\sin(3x)) + Ce^{2x}$ ans.
Q.6)	Solve the D.E. $\cos^2 x\frac{dy}{dx} + y = \tan x$
Sol.6)	Divide by $\cos^2 x$ $\Rightarrow \frac{dy}{dx} + y\sec^2 x = \tan x.\sec^2 x$ Here $P = \sec^2 x$ and $\theta = \tan x \sec^2 x$ $I.F. = e^{\int Pdx} = e^{\int \sec^2 x dx} = e^{\tan x}$ $\therefore I.F. = e^{\tan x}$ Solving is giving by $y.(I.F.) = \int \theta\,(I.F.)dx + C$ $\Rightarrow y.e^{\tan x} = \int \tan x.\sec^2 x.e^{\tan x}dx + C$ Put $\tan x = t$ $\Rightarrow \sec^2 x\;dx = dt$

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	$\therefore y.e^{\tan x} = \int e^t.t\,dt + C$ $\Rightarrow ye^{\tan x} = t.e^t - \int e^t dt + C$ $\Rightarrow ye^{\tan x} = te^t - e^t + C$ $\Rightarrow ye^{\tan x} = e^{\tan x}(\tan x - 1) + C$ ans.
Q.7)	Solve the D.E. $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$
Sol.7)	Divide by (x^2+1) $\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{\sqrt{x^2+4}}{x^2+1}$ Comparing with $\frac{dy}{dx} + Py = \theta$ We have, $P = \frac{2x}{x^2+1}$ and $\theta = \frac{\sqrt{x^2+4}}{x^2+1}$ $I.F. = e^{\int P dx} = e^{\int\left(\frac{2x}{x^2+1}\right)dx}$ Put $x^2+1 = t \Rightarrow 2xdx = dt$ $I.F. = e^{\int\frac{dt}{t}} = e^{\log t} = t = x^2+1$ $\therefore I.F. = x^2+1$ Solution is given by $y.(I.F.) = \int \theta\,(I.F.)dx + C$ $\Rightarrow y.(x^2+1) = \int \frac{\sqrt{x^2+4}}{x^2+1}.(x^2+1)dx + C$ $\Rightarrow y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + \frac{4}{2}\log\lvert x + \sqrt{x^2+4}\rvert + C$ $\Rightarrow y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + 2\log\lvert x + \sqrt{x^2+4}\rvert + C$
Q.8)	Find the particular solution of the D.E. $\frac{dy}{dx} + y = \cos x - \sin x.$ given $y(0) = 2$
Sol.8)	Comparing with $\frac{dy}{dx} + Py = \theta$ Where $P = 1$ and $\theta = \cos x - \sin x$ $I.F. = e^{\int P dx} = e^{\int 1.dx} = e^x \Rightarrow I.F. = e^x$ Solution is given by $y.(I.F.) = \int \theta\,(I.F.)dx + C$ $\Rightarrow ye^x = \int e^x.(\cos x - \sin x)\,dx + C$ $\Rightarrow ye^x = \int e^x.\cos x\,dx - \int e^x \sin x\,dx + C$ $\Rightarrow ye^x = \cos x\,e^x - \int -\sin x.e^x - \int e^x \sin x\,dx - \int e^x \sin x\,dx + C$ $\Rightarrow ye^x = e^x \cos x \rightarrow \int e^x \sin x\,dx - \int e^x \sin x\,dx + C$ $ye^x = e^x \cos x + C$ Put in Initial condition i.e., $x = 0$ and $y = 2$ $\Rightarrow 2e^{\circ} = e^{\circ}.cos + C$ $\Rightarrow 2 = C$ $\therefore ye^x = e^x \cos x + 2$ (or) $y = \cos x + 2e^x$ is the particular solution
Q.9)	Find the general solution of the

	$\frac{dy}{dx} + x\sin(2y) = x^3\, cos^2\, y$
Sol.9)	We have $\frac{dy}{dx} + x\sin(2y) = x^3 cos^2\, y$ Divide by $cos^2 y$ $\Rightarrow sec^2\, y\, \frac{dy}{dx} +\ x\sin(24) = x^3$ $\Rightarrow sec^2\, y\, \frac{dy}{dx} + x.\frac{2\sin y \cos y}{cos^2\, y} = x^3$ $\Rightarrow sec^2\, y\, \frac{dy}{dx} + 2x\tan y = x^3$ Let $\tan y = V \Rightarrow sec^2\, y\, \frac{dy}{dx} = \frac{dv}{dx}$ $\therefore \frac{dv}{dx} + 2xv = x^3$ This a linear D.E of the form $\frac{dv}{dx} + pv = Q$ Here $p = 2x\ and$
Q.10)	Find the particular solution of the D.E. $\frac{dy}{dx} + y = \cos x - \sin x$ given $y(0) = 2$.
Sol.10)	Compare with $\frac{dy}{dx} + Py = \theta$. We have, $P = 1$ and $\theta = \cos x - \sin x$ $I.F. = e^{\int P dx} = e^{\int 1 dx} = e^x \Rightarrow I.F. = e^x$ Solution is given by $y.(I.F.) = \int \theta\,(I.F.)dx + C$ $\Rightarrow ye^x = \int e^x.(\cos x\,.\sin x)dx + C$ $\Rightarrow ye^x = \int e^x \cos x\ \ dx - \int e^x \sin x\, dx + C$ $\Rightarrow ye^x = \cos x\, e^x - \sin x\ \ dx + C$ $\Rightarrow ye^x = \cos x + \int e^x \sin x\, dx - \int e^x \sin x\, dx + C$ $\Rightarrow ye^x = e^x \cos x + C$ Put initial condition i.e., $x = 0$ & $y = 2$ $\Rightarrow 2e^0 = e^0.\cos 0 + C$ $\Rightarrow 2 = C$ $\therefore ye^x = e^x.\cos x + 2$ Or $y = \cos x + 2e^{-x}$ is the particular solution ans.