INTERNATIONAL INDIAN SCHOOL, RIYADH

Work sheet XII mathematics

Inverse Trigonometric Functions. I Term

1. Prove that $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1} 3 = \frac{\pi}{4}$.

2. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$, prove that

$a + b + c = abc$.

3. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, ~~p.t~~ Prove that

$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$

4. Write in the simplest form

$\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

5. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$.

6. Solve for x:

$\tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7)$.

7. Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$.

8. Prove that $\cos\left[\tan^{-1}\{\sin(\cot^{-1} x)\}\right] = \sqrt{\frac{1+x^2}{2+x^2}}$

9. Prove that $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\left(\frac{1}{99}\right) = \frac{\pi}{4}$.

10. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, find x.

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Assignment

XII Mathematics I Term

Worksheet on matrices and Determinants

class: 12th

) Find a, b, c and d from $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$

) Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+2j)^2}{2}$

) Calculate

i) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Find X if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

) If $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$ find x, y, z and w

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ then show that $A^3 - 23A - 40I = 0$

) Find k if $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $A^2 = kA - 2I$ where I is 2×2 identity matrix

) Express $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix

If $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$; $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ verify $(AB)' = B'A'$

Express $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.

Using elementary operations find A^{-1} if it exists

a) $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

c) $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

12) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$; $\forall n \in N$

13) If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ show that $(aI + bA)^n = a^n I + n \cdot a^{n-1} bA$ where I is 2×2 unit matrix and $\forall n \in N$

14) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$; $n \in N$

15) If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}$ prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$; $\forall n \in N$

16) If the matrix A is both symmetric and skew symmetric then A must be?

17) Find "x" if $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

18) If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then prove that $1 + xyz = 0$

19) Show that (use properties) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab$

20) Using properties of determinants prove that

i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

ii) $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

iii) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

iv) $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$

v) $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = 0$

vi) $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

vii) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

viii) $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$

ix) $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

x) $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$

21) Solve:

i) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

ii) $x+y+z=6$; $3z+x=11$ and $x+z=2y$

iii) $x-y+2z=7$; $3x+4y-5z=-5$ and $2x-y+3z=12$

22) If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Using A^{-1} solve $2x-3y+5z=11$; $3x+2y-4z=-5$; $x+y-2z=-3$

23) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ find A^{-1} using A^{-1} solve $x+2y+z=4$; $-x+y+z=0$; $x-3y+z=2$

24) For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ show that $A^3-6A^2+5A+11I=0$ hence find A^{-1}

25) A square matrix A of order 3×3 has det A as 4 find det(3A)

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