

Integration (Indefinite Integrals)

Q.1)	(i) $I = \int \sin^2 x \, dx$ (iii) $I = \int \cot^2(3x) \, dx$	(ii) $I = \int \tan^2(2x) \, dx$ (iv) $I = \int \cos^2(4x) \, dx$
Sol.1)	<p>(i) $I = \int \sin^2 x \, dx$ $= \frac{1}{2} \int 1 - \cos(2x) \, dx$ $= \frac{1}{2} \left[x - \frac{\sin(2x)}{2} \right] + c$ ans.</p> <p>(ii) $I = \int \tan^2(2x) \, dx$ $= \int \sec^2(2x) - 1 \, dx$ $= \frac{\tan(2x)}{2} - x + c$ ans.</p> <p>(iii) $I = \int \cot^2(3x) \, dx$ $= \int \operatorname{cosec}^2(3x) - 1 \, dx$ $= \frac{-1}{3} \cot(3x) - x + c$ ans.</p> <p>(iv) $I = \int \cos^2(4x) \, dx$ $= \frac{1}{2} \int 1 + \cos(8x) \, dx$ $= \frac{1}{2} \left[x + \frac{\sin(8x)}{8} \right] + c$ ans.</p>	
Q.2)	(i) $I = \int \sin^3 x \, dx$	(ii) $I = \int \cos^3(2x) \, dx$
Sol.2)	<p>(i) $I = \int \sin^3 x \, dx$ $= \frac{1}{4} \int 3\sin x - \sin(3x) \, dx$ $= \frac{1}{4} \left[-3\cos x + \frac{\cos(3x)}{3} \right] + c$ ans.</p> <p>(ii) $I = \int \cos^3(2x) \, dx$ $= \frac{1}{4} \int 3\cos(2x) + \cos(6x) \, dx$ $= \frac{1}{4} \left[\frac{3}{2} \sin(2x) + \frac{1}{6} \sin(6x) \right] + c$ ans.</p>	
Q.3)	(i) $I = \int \tan^3 x \, dx$	(ii) $I = \int \cot^3(3x) \, dx$
Sol.3)	<p>(i) $I = \int \tan^3 x \, dx$ $= \int \tan x \cdot \tan^2 x \, dx$ $= \int \tan x \cdot (\sec^2 x - 1) \, dx$ $= \int \tan x \cdot \sec^2 x - \tan x \, dx$ $= \int \tan x \cdot \sec^2 x \, dx - \int \tan x \, dx$ put $\tan x = t$ in (I)...</p>	

	$\sec^2 x \, dx = dt$ $\therefore I = \int t \, dt - \log \sec x $ $= \frac{t^2}{2} - \log \sec x + c$ <p>Replacing t by tan x</p> $I = \frac{1}{2} \tan^2 x - \log \sec x + c \quad \text{ans.}$ (ii) $I = \int \cot^3(3x) \, dx$ $= \int \cot(3x) \cdot \cot^2(3x) \, dx$ $= \int \cot(3x) (\operatorname{cosec}^2(3x) - 1) \, dx$ $= \int \cot(3x) \cdot \operatorname{cosec}^2(3x) - \cot(3x) \, dx$ $= \int \cot(3x) \cdot \operatorname{cosec}^2(3x) \, dx - \int \cot(3x) \, dx$ <p>put $\cot(3x) = t$ in (I)</p> $\therefore -\operatorname{cosec}^2(3x) \cdot 3 \, dx = dt$ $\operatorname{cosec}^2(3x) \, dx = \frac{-dt}{3}$ $\therefore I = -\frac{1}{3} \int t \, dt - \frac{1}{3} \log \sin(3x) $ $= -\frac{1}{6} t^2 - \frac{1}{3} \log \sin(3x) + c$ <p>Replacing t</p> $I = -\frac{1}{6} \cot^2(3x) - \frac{1}{3} \log \sin(3x) + c \quad \text{ans.}$
Q.4)	(i) $I = \int \sin^4 x \, dx$ (ii) $I = \int \cos^4(2x) \, dx$
Sol.4)	(i) $I = \int \sin^4 x \, dx$ $= \int (\sin^2 x)^2 \, dx$ $= \int \left(\frac{1 - \cos(2x)}{2} \right)^2 \, dx$ $= \frac{1}{4} \int 1 + \cos^2(2x) - 2\cos(2x) \, dx$ $= \frac{1}{4} \int 1 + \frac{1 + \cos(4x)}{2} - 2\cos(2x) \, dx$ $= \frac{1}{8} \int 2 + 1 + \cos(4x) - 4\cos(2x) \, dx$ $= \frac{1}{8} \int 3 + \cos(4x) - 4\cos(2x) \, dx$ $I = \frac{1}{8} \left[3x + \frac{\sin(4x)}{4} - 2\sin(2x) \right] + c \quad \text{ans.}$ (ii) $I = \int \cos^4(2x) \, dx$ $= \int (\cos^2(2x))^2 \, dx$ $= \int \left(\frac{1 + \cos(4x)}{2} \right)^2 \, dx$ $= \frac{1}{4} \int 1 + \cos^2(4x) + 2\cos(4x) \, dx$ $= \frac{1}{4} \int 1 + \frac{1 + \cos(8x)}{2} + 2\cos(4x) \, dx$ $= \frac{1}{8} \int 3 + \cos(8x) + 4\cos(4x) \, dx$

	$= I = \frac{1}{8} \left[3x + \frac{\sin(8x)}{8} + \frac{4}{4} \sin(4x) \right] + c$ ans.
Q.5)	(i) $I = \int \tan^4 x dx$ (ii) $I = \int \cot^4(3x) dx$
Sol.5)	<p>(i) $I = \int \tan^4 x dx$ $= \int \tan^2 x \cdot \tan^2 x dx$ $= \int \tan^2 x \cdot (\sec^2 x - 1) dx$ $= \int \tan^2 x \cdot \sec^2 x dx - \int \tan^2 x dx$ $= \int \tan^2 x \cdot \sec^2 x dx - \int \sec^2 x - 1 dx$ put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $I = \int t^2 dt - (\tan x - x)$ $= \frac{t^3}{3} - \tan x + x + c$ $I = \frac{1}{3} \tan^3 x - \tan x + x + c$ ans.</p> <p>(ii) $I = \int \cot^4(3x) dx$ $= \int \cot^2(3x) \cdot \cot^2(3x) dx$ $= \int \cot^2(3x) \cdot (\operatorname{cosec}^2(3x) - 1) dx$ $= \int \cot^2(3x) \cdot \operatorname{cosec}^2(3x) dx - \int \cot^2(3x) dx$ put $\cot(3x) = t$ $-3 \operatorname{cosec}^2(3x) dx = dt$ $\therefore \operatorname{cosec}^2(3x) dx = -\frac{dt}{3}$ $I = -\frac{1}{3} \int t^2 dt - \int \operatorname{cosec}^2(3x) - 1 dx$ $= -\frac{1}{3} \cdot \frac{t^3}{3} - \left(\frac{\cot(3x)}{3} - x \right) + c$ $I = -\frac{1}{9} \cot^2(3x) - \frac{\cot(3x)}{3} + x + c$ ans.</p>
Q.6)	$I = \int \sec^4 x dx$
Sol.6)	<p>$I = \int \sec^4 x dx$ $= \int \sec^2 x \cdot \sec^2 x dx$ $= \int (1 + \tan^2 x) \sec^2 x dx$ $= \int \sec^2 x dx + \int \tan^2 x \cdot \sec^2 x dx$ put $\tan x = t$ $\therefore \sec x dx = dt$ $= \tan x + \int t^2 dt$ $= \tan x + \frac{t^3}{3} + c$ $= I = \tan x + \frac{\tan^3}{3} + c$ ans.</p>
→ <u>Sin x and Cos x in multiplication with same power :-</u>	
Q.7)	(i) $I = \int \sin^2 x \cdot \cos^2 x dx$ (ii) $I = \int \sin^4 x \cdot \cos^4 x dx$

Sol.7)	<p>(i) $I = \int \sin^2 x \cdot \cos^2 x dx$</p> $= \int (\sin x \cdot \cos x)^2 dx$ $= \int \left(\frac{\sin(2x)}{2}\right)^2 dx$ $= \frac{1}{4} \int \sin^2(2x) dx$ $= \frac{1}{4} \int \frac{1 - \cos(4x)}{2} dx$ $= \frac{1}{8} \int 1 - \cos(4x) dx$ $= I = \frac{1}{8} \left[x - \frac{\sin(4x)}{4} \right] + c \quad \text{ans.}$ <p>(ii) $I = \int \sin^4 x \cdot \cos^4 x dx$</p> $= \int (\sin x \cdot \cos x)^4 dx$ $= \int \left(\frac{\sin(2x)}{2}\right)^4 dx$ $= \frac{1}{16} \int \sin^4(2x) dx$ $= \frac{1}{16} \int (\sin^2(2x))^2 dx$ $= \frac{1}{16} \int \left(\frac{1 - \cos(4x)}{2}\right)^2 dx$ $= \frac{1}{64} \int 1 + \cos^2(4x) - 2\cos(4x) dx$ $= \frac{1}{64} \int 1 + \frac{1 + \cos(8x)}{2} - 2\cos(4x) dx$ $= \frac{1}{128} \int 3 + \cos(8x) - 4\cos(4x) dx$ $= I = \frac{1}{128} \left[3x + \frac{\sin(8x)}{8} - \sin(4x) \right] + c \quad \text{ans.}$
→	<u>Sin x and Cos x in multiplication with different power :-</u>
Q.8)	(i) $I = \int \sin^3 x \cdot \cos^4 x dx$
Sol.8)	<p>(i) $I = \int \sin^3 x \cdot \cos^4 x dx$</p> $= \int \sin^2 x \cdot \cos^4 x \cdot \sin x dx$ $= \int (1 - \cos^2 x) \cdot \cos^4 x \cdot \sin x dx$ <p>put $\cos x = t$</p> $\therefore -\sin x dx = dt$ $\sin x dx = -dt$ $I = -\int (1 - t^2)t^4 dt$ $= -\int t^4 - t^6 dt$ $= -\left[\frac{t^5}{5} - \frac{t^7}{7}\right] + c$ $I = -\left[\frac{\cos^5 x}{5} - \frac{\cos^7 x}{7}\right] + c \quad \text{ans.}$
Q.9)	(i) $I = \int \sin^3 x \cdot \cos^5 x dx$ (ii) $I = \int \sin^5 x dx$ (iii) $I = \int \cos^7 x dx$
Sol.9)	(i) $I = \int \sin^3 x \cdot \cos^5 x dx$

	$= \int \sin^2 x \cdot \cos^5 x \cdot \sin x dx$ <p>put $\cos x = t$ $\sin x dx = -dt$</p> $\therefore I = -\int (1 - t^2)t^5 dt$ $= -\int t^5 - t^7 dt$ $= \frac{-t^6}{6} + \frac{t^8}{8} + c$ $= \frac{-\cos^6 x}{6} + \frac{\cos^8 x}{8} + c \quad \text{ans.}$ (ii) $I = \int \sin^5 x dx$ $= \int \sin^4 x \cdot \sin x dx$ $= \int (1 - \cos^2 x)^2 \cdot \sin x dx$ <p>put $\cos x = t$ $\sin x dx = -dt$</p> $\therefore I = -\int (1 - t^2)^2 dt$ $= -\int 1 + t^4 - 2t^2 dt$ $= -\left[t + \frac{t^5}{5} - 2\frac{t^3}{3} \right] + c$ $I = -\left[\cos x + \frac{\cos^5 x}{5} - \frac{2\cos^3 x}{3} \right] + c \quad \text{ans.}$ (iii) $I = \int \cos^7 x dx$ $= \int \cos^6 x \cdot \cos x dx$ $= \int (\cos^2 x)^3 \cdot \cos x dx$ $= \int (1 - \sin^2 x)^3 \cos x dx$ <p>put $\sin x = t$ $\cos x dx = dt$</p> $\therefore I = \int (1 - t^2)^3 dt$ $= \int 1 - t^6 - 3t^2 + 3t^4 dt$ $= t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5} + c$ $= I = \sin x - \frac{\sin^7 x}{7} - \sin^3 x + \frac{3}{5} \sin^5 x + c \quad \text{ans.}$
Q.10)	$I = \int \cos^7 x dx$
Sol.10)	$I = \int \cos^7 x dx$ $= \int \cos^6 x \cdot \cos x dx$ $= \int (\cos^2 x)^3 \cdot \cos x dx$ $= \int (1 - \sin^2 x)^3 \cos x dx$ <p>put $\sin x = t$ $\cos x dx = dt$</p> $\therefore I = \int (1 - t^2)^3 dt$ $= \int 1 - t^6 - 3t^2 + 3t^4 dt$ $= t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5} + c$

$= I = \sin x - \frac{\sin^7 x}{7} - \sin^7 x + \frac{3}{5} \sin^5 x + c \quad \text{ans.}$

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