

Integration (Indefinite integral)

Q.1)	$I = \int \frac{x^2}{(a+bx)^2} dx$
Sol.1)	$I = \int \frac{x^2}{(a+bx)^2} dx$ <p>put $a+bx = t$ $b dx = dt \Rightarrow dx = \frac{dt}{b}$</p> $\therefore I = \frac{1}{b} \int \frac{x^2}{t^2} dt$ $= \frac{1}{b} \int \frac{\left(t - \frac{a}{b}\right)^2}{t^2} dt$ $= \frac{1}{b} \cdot \frac{1}{b^2} \int \frac{t^2 + a^2 - 2at}{t^2} dt$ <p>Separate</p> $= \frac{1}{b^3} \int 1 + \frac{a^2}{t^2} - \frac{2a}{t} dt$ $= \frac{1}{b^3} \left[t - \frac{a^2}{t} - 2a \log t \right] + c$ $= \frac{1}{b^3} \left[(a+bx) - \frac{a^2}{a+bx} - 2a \log a+bx \right] + c \quad \text{ans.}$
→ Type: When degree of Numerator \geq degree of Denominator then divide and write $\int \frac{N}{D} dx = \int \theta + \frac{R}{D} dx$	
Q.2)	(i) $I = \int \frac{x^7}{x-1} dx$ (ii) $I = \int \frac{1}{x^{1/2} + x^{1/3}} dx$
Sol.2)	<p>(i) $I = \int \frac{x^7}{x-1} dx$ clearly degree of $N^r >$ degree of D^r (then divide) $\therefore I = \int \theta + \frac{R}{D} dx$</p> $= \int (x^6 - x^5 + x^4 - x^3 + x^2 - x + 1) - \frac{1}{x+1} dx$ $= \int \frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x$ $= -\log x+1 + c \quad \text{ans.}$ <p>(ii) $I = \int \frac{1}{x^{1/2} + x^{1/3}} dx$ put $x = t^6$ {L.C.M of 2 & 3 = 6} $dx = 6t^5 dt$ $\therefore I = 6 \int \frac{t^5 dt}{t^3 + t^2}$</p> $= \int \frac{t^5}{t^2(t+1)} dt$ $= \int \frac{t^3}{t+1} dt$ <p>Degree of $N >$ degree of D (then divide)</p> $= \int (t^2 - t + 1) - \frac{1}{t+1} dt$

	$I = \frac{t^3}{3} - \frac{t^2}{2} + t - \log t + 1 + c$ <p>replacing t by $x^{1/6}$</p> $\therefore I = \frac{x^{1/2}}{3} - \frac{x^{1/3}}{2} + x^{1/6} - \log x^{1/6} + 1 + c \quad \text{ans.}$
Q.3)	$(i) I = \int \frac{e^{2x}-1}{e^{2x}+1} dx \quad (ii) I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx \quad (iii) I = \int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx$
Sol.3)	<p>(i) $I = \int \frac{e^{2x}-1}{e^{2x}+1} dx$ take e^x common in N and D $= \int \frac{e^x(e^x - e^{-x})}{e^x(e^x + e^{-x})} dx$ put $e^x + e^{-x} = t$ $\therefore (e^x - e^{-x})dx = dt$ $\therefore I = \int \frac{dt}{t}$ $= \log t + c$ $I = \log e^x - e^{-x} + c \quad \text{ans.}$</p> <p>(ii) $I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx$ Divide N and D by $\cos^2 x$ $I = \int \frac{\frac{\sqrt{\tan x}}{\cos^2 x}}{\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}} dx$ $= \int \frac{\sqrt{\tan x} \cdot \sec^2 x}{\tan x} dx$ put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $\therefore I = \int \frac{\sqrt{t}}{t} dt$ $= \int \frac{1}{\sqrt{t}} dt$ $I = 2\sqrt{t} + c$ $I = 2\sqrt{\tan x} + c \quad \text{ans.}$</p> <p>(iii) $I = \int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx$ put $2^{2^{2^x}} = t$ $\therefore 2^{2^{2^x}} \cdot \log 2 \cdot 2^{2^x} \cdot \log 2 \cdot 2^x \cdot \log 2 dx = dt$ $\Rightarrow 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x \cdot (\log 2)^3 dx = dt$ $\Rightarrow 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx = \frac{dt}{(\log 2)^3}$ $\therefore I = \frac{1}{(\log 2)^3} \int dt$ $= \frac{1}{(\log 2)^3} t + c$ $I = \frac{1}{(\log 2)^3} \cdot 2^{2^{2^x}} + c \quad \text{ans.}$</p>
Q.4)	$(i) I = \int \frac{x^5}{\sqrt{1+x^3}} dx \quad (ii) I = \int 5^{x+\tan^{-1}x} \cdot \left(\frac{x^2+1}{x^2+1}\right) dx$

	$(iii) I = \int \frac{e^{\sqrt{x}} \cdot \cos(e^{\sqrt{x}})}{\sqrt{x}} dx \quad (iv) I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$
Sol.4)	<p>(i) $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ $\int \frac{x^3 \cdot x^2}{\sqrt{1+x^3}} dx$ put $1+x = t$ $3x^2 dx = dt$ $x^2 dx = \frac{dt}{3}$ $\therefore I = \frac{1}{3} \int \frac{x^3}{\sqrt{t}} dt$ $= \frac{1}{3} \int \frac{(t-1)}{\sqrt{t}} dt$ Separate $= \frac{1}{3} \int \sqrt{t} - \frac{1}{\sqrt{t}} dt$ $= \frac{1}{3} \left[\frac{2}{3} t^{3/2} - 2\sqrt{t} \right] + c$ $= \frac{1}{3} \left[\frac{2}{3} (1+x^3)^{3/2} - 2\sqrt{(1+x^3)} \right] + c \quad \text{ans.}$</p> <p>(ii) $I = \int 5^{x+\tan^{-1}x} \cdot \left(\frac{x^2+1}{x^2+1} \right) dx$ Hint: put $x + \tan^{-1}x = t$ $1 + \frac{1}{1+x^2} dx = dt$ $\frac{5^{x+\tan^{-1}x}}{\log 5} + c \quad \text{ans.}$</p> <p>(iii) $I = \int \frac{e^{\sqrt{x}} \cdot \cos(e^{\sqrt{x}})}{\sqrt{x}} dx$ put $e^{\sqrt{x}} = t$ $\frac{e^{\sqrt{x}}}{2\sqrt{x}} dx = dt$ $e^{\sqrt{x}} \frac{dx}{\sqrt{x}} = 2dt$ $\therefore I = 2 \int \cos t dt$ $= 2 \sin t + c$ $= 2 \sin(e^{\sqrt{x}}) + c \quad \text{ans.}$</p> <p>(iv) $I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$ put $xe^x = t$ $(xe^x + e^x) dx = dt$ $e^x(x+1) dx = dt$ $\therefore I = \int \frac{dt}{\sin^2 t}$ $= \int \operatorname{cosec}^2 t dt$ $= -\cot t + c$ $= -\cot(xe^x) + c \quad \text{ans.}$</p>
Q.5)	<p>(i) $I = \int \frac{1}{1+\tan x} dx \quad (ii) I = \int \frac{1}{1+\cot x} dx$</p>

Sol.5)	$(i) I = \int \frac{1}{1+\tan x} dx$ $= \int \frac{1}{1+\frac{\sin x}{\cos x}} dx$ $= \int \frac{\cos x}{\cos x + \sin x} dx$ $= \frac{1}{2} \int \frac{2 \cos x}{\cos x + \sin x} dx$ $= \frac{1}{2} \int \frac{\cos x + \sin x + \cos x - \sin x}{\cos x + \sin x} dx$ $= \frac{1}{2} \int \frac{(\cos x + \sin x) + (\cos x - \sin x)}{\cos x + \sin x} dx$ <p>Separate</p> $= \frac{1}{2} \int 1 + \frac{\cos x - \sin x}{\cos x + \sin x} dx$ $= \frac{1}{2} \int 1 \cdot dx + \frac{1}{2} \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ <p>put $\cos x + \sin x = t$ $(-\sin x + \cos x) dx = dt$</p> $= \frac{1}{2} x + \frac{1}{2} \int \frac{dt}{t}$ $I = \frac{1}{2} x + \frac{1}{2} \log \cos x + \sin x + c \quad \text{ans.}$ $(ii) I = \int \frac{1}{1+\cot x} dx$ $= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$ $= \int \frac{\sin x}{\sin x + \cos x} dx$ $= \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx$ $= \frac{1}{2} \int \frac{\sin x + \sin x + \cos x - \cos x}{\sin x + \cos x} dx$ $= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{\sin x + \cos x} dx$ $= \frac{1}{2} \int 1 + \frac{\sin x - \cos x}{\sin x + \cos x} dx$ $= \frac{1}{2} x + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ <p>put $\sin x + \cos x = t$ $(\cos x - \sin x) dx = dt$</p> $I = \frac{1}{2} x + \frac{1}{2} \int \frac{dt}{t}$ $= \frac{1}{2} x + \frac{1}{2} \log \sin x + \cos x + c \quad \text{ans.}$
Q.6)	$(i) I = \int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx \quad (ii) I = \int (x^4 + 1)^{-1} \cdot e^{3 \log x} dx \quad (iii) I = \int e^{\log \sqrt{x}} dx$ $(iv) I = \int \frac{(a^x + b^x)}{a^x + b^x} dx$
Sol.6)	$(i) I = \int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx$ $= \int \frac{e^{\log x^5} - e^{\log x^4}}{e^{\log x^3} - e^{\log x^2}} dx$ $= \int \frac{x^5 - x^4}{x^3 - x^2} dx \quad \dots \dots \{ \because e^{\log x} = x \}$ $= \int \frac{x^4(x-1)}{x^2(x-1)} dx$

	$= \int x^2 dx$ $I = \frac{x^3}{3} + c \quad \text{ans.}$ (ii) $I = \int (x^4 + 1)^{-1} \cdot e^{3 \log x} dx$ $= \int \frac{e^{\log x^3}}{x^4 + 1} dx$ $= \int \frac{x^3}{x^4 + 1} dx$ put $x^4 + 1 = t$ $4x^3 dx = dt \Rightarrow x^3 dx = \frac{dt}{4}$ $\therefore I = \frac{1}{4} \int \frac{dt}{t}$ $= \frac{1}{4} \log x^4 + 1 + c \quad \text{ans.}$ (iii) $I = \int e^{\log \sqrt{x}} dx$ $= \int \frac{\sqrt{x}}{x} dx$ $= \int \frac{1}{\sqrt{x}} dx$ $= 2\sqrt{x} + c \quad \text{ans.}$ (iv) $I = \int \frac{(a^x + b^x)}{a^x b^x} dx$ $= \int \frac{a^{2x} + b^{2x} + 2a^x \cdot b^x}{a^x b^x} dx$ Separate $= \int \frac{a^{2x}}{a^x b^x} + \frac{b^{2x}}{a^x b^x} + \frac{2a^x b^x}{a^x b^x} dx$ $= \int \frac{a^x}{b^x} + \frac{b^x}{a^x} + 2 dx$ $= \int \left(\frac{a}{b}\right)^x + \left(\frac{b}{a}\right)^x + 2 dx$ $= I = \frac{\left(\frac{a}{b}\right)^x}{\log(a/b)} + \frac{\left(\frac{b}{a}\right)^x}{\log(b/a)} + 2x + c \quad \text{ans.}$
Q.7)	(i) $I = \int (2 \tan x - 3 \cot x)^2 dx$ (ii) $I = \int \frac{1}{\sin^2 x \cdot \cos^2 x} dx$ (iii) $I = \int \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}} dx$ (iv) $I = \int \frac{\sin(2x)}{a^2 \sin^2 + b^2 \cos^2 x} dx$
Sol.7)	(i) $I = \int (2 \tan x - 3 \cot x)^2 dx$ $= \int 4 \tan^2 x + 9 \cot^2 x - 12 \tan x \cdot \cot x dx$ $= \int 4(\sec^2 x - 1) + 9(\operatorname{cosec}^2 x - 1) - 12 dx$ $= 4(\tan x - x) + 9(-\cot x - x) - 12x + c$ $= 4 \tan x - 9 \cot x - 25x + c \quad \text{ans.}$ (ii) $I = \int \frac{1}{\sin^2 x \cdot \cos^2 x} dx$ $= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$ Separate: $= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$

	$= \int \sec^2 x + \operatorname{cosec}^2 x dx$ $= I = \tan x - \cot x + c \quad \text{ans.}$ $(iii) I = \int \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}} dx$ $= \int \tan^{-1} \sqrt{\frac{1-\cos(\frac{\pi}{2}-x)}{1+\cos(\frac{\pi}{2}+x)}} dx$ $= \int \tan^{-1} \sqrt{\frac{2\sin^2(\frac{\pi}{4}-\frac{x}{2})}{2\cos^2(\frac{\pi}{4}+\frac{x}{2})}} dx$ $= \int \tan^{-1} \sqrt{\tan^2(\frac{\pi}{4}-\frac{x}{2})} dx$ $= \int \tan^{-1} \left(\tan(\frac{\pi}{4}-\frac{x}{2}) \right) dx$ $= \int \frac{\pi}{4} - \frac{x}{2} dx$ $I = \frac{\pi x}{4} - \frac{x^2}{4} + c \quad \text{ans.}$ $(iv) I = \int \frac{\sin(2x)}{a^2 \sin^2 x + b^2 \cos^2 x} dx$ <p>put $a^2 \sin^2 x + b^2 \cos^2 x = t$</p> $a^2 2 \sin x \cos x - b^2 2 \cos x \sin x dx = dt$ $a^2 \sin(2x) - b^2 \sin(2x) dx = dt$ $\sin(2x)(a^2 - b^2) dx = dt$ $\sin(2x) dx = \frac{dt}{a^2 - b^2}$ $\therefore I = \frac{1}{a^2 - b^2} \int \frac{dt}{t}$ $= \frac{1}{a^2 - b^2} \log t + c$ $= I = \frac{1}{a^2 - b^2} \log a^2 \sin^2 x + b^2 \cos^2 x + c \quad \text{ans.}$
Q.8)	$(i) I = \int \frac{\log(\tan \frac{x}{2})}{\sin x} dx$ $(ii) \text{ If } f'(x) = x + b, f(1) = 5 \text{ and } f(2) = 13. \text{ Find } f(x)$ $(iii) \text{ If } f'(x) = 3x^2 - \frac{2}{x^3} \text{ and } f(1) = 0. \text{ Find } f(x).$ $(iv) I = \int \frac{e^{x-1} + e^{-1}}{e^x + x^e} dx$
Sol.8)	$i) I = \int \frac{\log(\tan \frac{x}{2})}{\sin x} dx$ <p>put $\log(\tan \frac{x}{2}) = t$</p> $\frac{1}{\tan \frac{x}{2}} \cdot \sec^2 \left(\frac{x}{2} \right) \cdot \frac{1}{2} dx = dt$ $\Rightarrow \frac{1}{\frac{\cos(\frac{x}{2})}{\sin(\frac{x}{2})}} \cdot \frac{1}{2} dx = dt$ $\Rightarrow \frac{1}{2 \sin(\frac{x}{2}) \cos(\frac{x}{2})} dx = dt$

	$\Rightarrow \frac{1}{\sin x} dx = dt$ $\therefore I = \int t dt$ $= \frac{t^2}{2} + c$ $= \frac{\left(\log\left(\tan\frac{x}{2}\right)\right)^2}{2} + c \quad \text{ans.}$ <p>(ii) We have, $f(x) = \int f'(x) dx$</p> $f(x) = \int (x + b) dx$ $f(x) = \frac{x^2}{2} + bx + c$ $f(1) = 5 \text{ and } f(2) = 13 \quad \dots \text{(given)}$ $5 = \frac{1}{2} + b + c$ $\frac{9}{2} = b + c \quad \dots (1)$ <p>and $13 = 2 + 2b + c$</p> $11 = 2b + c \quad \dots (2)$ <p>solving (1) & (2)</p> $b = \frac{13}{2} \text{ and } c = -2$ $\therefore f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2 \quad \text{ans.}$ <p>(iii) $f(x) = x^3 + \frac{1}{x^2} + c \quad \text{ans.}$</p> <p>(iv) $I = \int \frac{e^{x-1} + e^{-1}}{e^x + x^e} dx$</p> <p>put $e^x + x^e = t$</p> $e^x + ex^{e-1} dx = dt$ $\Rightarrow e(e^{x-1} + x^{e-1}) dx = dt$ $\Rightarrow e^{x-1} + x^{e-1} dx = \frac{dt}{e}$ $\therefore I = \frac{1}{e} \int \frac{dt}{t}$ $= \frac{1}{e} \log t + c$ $I = \frac{1}{e} \log e^x + x^e + c \quad \text{ans.}$
Q.9)	$(i) I = \int \frac{1}{x+\sqrt{x}} dx \quad (ii) I = \int \frac{(x+1)e^x}{\cos^2(xe^x)} dx \quad (iii) I = \int \frac{x^5}{\sqrt{1+x^3}} dx$
Sol.9)	$(i) I = \int \frac{1}{x+\sqrt{x}} dx$ $= \int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx$ <p>put $\sqrt{x} + 1 = t$</p> $\frac{1}{2\sqrt{x}} dx = dt$ $\frac{1}{\sqrt{x}} dx = 2dt$ $\therefore I = 2 \int \frac{dt}{t}$ $= 2 \log \sqrt{x} + 1 + c \quad \text{ans.}$

	$(ii) I = \int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$ <p>put $xe^x = t$ $(xe^x + e^x)dx = dt$ $e^x(x+1)dx = dt$</p> $\therefore I = \int \frac{dt}{\cos^2 t}$ $= \int \sec^2 t dt$ $= \tan t + c$ $= \tan(xe^x) + c \quad \text{ans.}$ $(iii) I = \int \frac{x^5}{\sqrt{1+x^3}} dx$ $= \int \frac{x^3 \cdot x^2}{\sqrt{1+x^3}} dt$ <p>put $1+x^3 = t^2 \quad \dots(1)$ $3x^2 dx = 2t dt$ $x^2 dx = \frac{2t}{3} dt$</p> $\therefore I = \frac{2}{3} \int \frac{x^3 \cdot t}{t} dt$ $= \frac{2}{3} \int (t^2 - 1) dt \quad \dots\dots(\text{from (1)})$ $= \frac{2}{3} \left(\frac{t^3}{3} - t \right) + c$ $I = \frac{2}{3} \left[\frac{(1+x^3)^{3/2}}{3} - (1+x^3)^{1/2} \right] + c \quad \text{ans.}$
Q.10)	$(i) I = \int \frac{1}{16-9x^2} dx \quad (ii) I = \int \frac{1}{\sqrt{16-9x^2}} dx \quad (iii) I = \int \frac{1}{4+9x^2} dx$ $(iv) I = \int \frac{1}{\sqrt{4+9x^2}} dx \quad (v) I = \int \frac{1}{9x^2-4} dx$
Sol.10)	$(i) I = \int \frac{1}{16-9x^2} dx$ $= \frac{1}{9} \int \frac{1}{\left(\frac{4}{3}\right)^2 - x^2} dx$ $= \frac{1}{9} \times \frac{1}{2 \times 4} \log \left \frac{\frac{4}{3}+x}{\frac{4}{3}-x} \right + c \quad \dots\dots \int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log$ $= \frac{1}{24} \log \left \frac{4+3x}{4-3x} \right + c \quad \text{ans.}$ $(ii) I = \int \frac{1}{\sqrt{16-9x^2}} dx$ $= \frac{1}{3} \int \frac{1}{\sqrt{\left(\frac{4}{3}\right)^2 - x^2}} dx$ $= \frac{1}{3} \sin^{-1} \left(\frac{x}{\frac{4}{3}} \right) + c \quad \dots\dots \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c$ $= \frac{1}{3} \sin^{-1} \left(\frac{3x}{4} \right) + c \quad \text{ans.}$ $(iii) I = \int \frac{1}{4+9x^2} dx$ $= \frac{1}{9} \int \frac{1}{\left(\frac{2}{3}\right)^2 + x^2} dx$



	$= \frac{1}{9} \times \frac{1}{\frac{2}{3}} \tan^{-1} \left(\frac{x}{\frac{2}{3}} \right) + c$ $= \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right) + c \quad \text{ans.}$
	$(iv) I = \int \frac{1}{\sqrt{4+9x^2}} dx$ $= \frac{1}{3} \int \frac{1}{\sqrt{\left(\frac{2}{3}\right)^2 + x^2}} dx$ $= \frac{1}{3} \log \left x + \sqrt{\left(\frac{2}{3}\right)^2 + x^2} \right + c \quad \text{ans.}$
	$(v) I = \int \frac{1}{9x^2-4} dx$ $= \frac{1}{9} \int \frac{1}{x^2 - \left(\frac{2}{3}\right)^2} dx$ $= \frac{1}{9} \times \frac{1}{\frac{2 \times 2}{3}} \log \left \frac{x - \frac{2}{3}}{x + \frac{2}{3}} \right + c$ $= \frac{1}{12} \log \left \frac{3x-2}{3x+2} \right + c \quad \text{ans.}$