



## Integration (Indefinite Integrals)

Q.1)	(i) $I = \int \frac{\cos(2x)-\cos(2\alpha)}{\cos x-\cos \alpha} dx$ (ii) $I = \int \frac{1+\cos(4x)}{\cot x-\tan x} dx$
Sol.1)	$  \begin{aligned}  \text{(i)} \quad & I = \int \frac{\cos(2x)-\cos(2x)}{\cos x-\cos x} dx \\  &= \int \frac{(2\cos^2 x-1)-(2\cos^2 \alpha-1)}{\cos x-\cos \alpha} dx \\  &= 2 \int \frac{\cos^2 x-\cos^2 \alpha}{\cos x-\cos \alpha} dx \\  &= 2 \int \frac{(\cos x+\cos \alpha)(\cos x-\cos \alpha)}{\cos x-\cos \alpha} dx \\  &= I = 2[\sin x + x \cos \alpha] + c \quad \text{ans.}  \end{aligned}  $ $  \begin{aligned}  \text{(ii)} \quad & I = \int \frac{1+\cos(4x)}{\cot x-\tan x} dx \\  &= \int \frac{2\cos^2(2x)}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx \\  &= \int \frac{2\cos^2(2x)}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx \\  &= \int \frac{2\sin x \cdot \cos x \cdot \cos^2(2x)}{\cos^2 x - \sin^2 x} dx \\  &= \int \frac{\sin(2x) \cdot \cos^2(2x)}{\cos(2x)} dx \\  &= \int \sin(2x) \cdot \cos(2x) dx \\  &= \frac{1}{2} \int 2\sin(2x) \cdot \cos(2x) dx \\  &= \frac{1}{2} \int \sin(4x) dx \\  &= \frac{1}{2} \left( -\frac{\cos(4x)}{4} \right) + c \\  &I = -\frac{1}{8} \cos(4x) + c \quad \text{ans.}  \end{aligned}  $
→	<b>Type : Rationalize :</b> $\int \frac{1}{1 \pm \sin x} dx, \int \frac{1}{1 \pm \cos x} dx$
Q.2)	(i) $I = \int \frac{1}{1+\sin x} dx$ (ii) $I = \int \frac{\sin x}{1-\sin x} dx$
Sol.2)	$  \begin{aligned}  \text{(i)} \quad & I = \int \frac{1}{1+\sin x} dx \\  &\text{Rationalize} \\  &= \int \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx \\  &= \int \frac{1-\sin x}{\cos^2 x} dx \\  &\text{Separate} \\  &I = \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x}{\cos^2 x} dx \\  &= \int \sec^2 x dx - \int \tan x \sec x dx \\  &I = \tan x - \sec x + c \quad \text{ans.}  \end{aligned}  $

	$(ii) I = \int \frac{\sin x}{1-\sin x} dx$ <p>Rationalize</p> $= \int \frac{\sin x(1+\sin x)}{(1-\sin x)(1+\sin x)} dx$ $= \int \frac{\sin x + \sin^2 x}{\cos^2 x} dx$ <p>Separate</p> $= \int \tan x \cdot \sec x + \tan^2 x dx$ $= \int \tan x \cdot \sec x + \sec^2 x - 1 dx$ $I = -\sec x + \tan x - x + c \quad \text{ans.}$
Q.3)	$(i) I = \int \frac{\cos x - \cos(2x)}{1-\cos x} dx \quad (ii) I = \int \tan^{-1} \sqrt{\frac{1-\cos(2x)}{1+\cos(2x)}} dx$ $(iii) I = \int \tan^{-1}(\sec x + \tan x) dx$
Sol.3)	$(i) I = \int \frac{\cos x - \cos(2x)}{1-\cos x} dx$ $= \int \frac{\cos x - (2\cos^2 x - 1)}{1-\cos x} dx$ $= -\int \frac{2\cos^2 x - \cos x - 1}{1-\cos x} dx$ $= -\int \frac{(2\cos x + 1)(\cos x - 1)}{1-\cos x} dx$ $= -\int \frac{(2\cos x + 1)(\cos x - 1)}{-(\cos x - 1)} dx$ $= \int 2\cos x + 1 dx$ $= I = 2\sin x + x + c \quad \text{ans.}$ $(ii) I = \int \tan^{-1} \sqrt{\frac{1-\cos(2x)}{1+\cos(2x)}} dx$ $= \int \tan^{-1} \sqrt{\frac{2\sin^2 x}{2\cos^2 x}} dx$ $= \int \tan^{-1}(\tan x) dx$ $= \int x dx$ $I = \frac{x^2}{2} + c \quad \text{ans.}$ $(iii) I = \int \tan^{-1}(\sec x + \tan x) dx$ $= \int \tan^{-1} \left( \frac{1}{\cos x} + \frac{\sin x}{\cos x} \right) dx$ $= \int \tan^{-1} \left( \frac{1+\sin x}{\cos x} \right) dx$ $= \int \tan^{-1} \left[ \frac{1+\cos(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)} \right] dx$



	$  \begin{aligned}  &= \int \tan^{-1} \left( \frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)} \right) dx \\  &= \int \tan^{-1} \left( \cot\left(\frac{\pi}{4}-\frac{x}{2}\right) \right) dx \\  &= \int \tan^{-1} \left[ \tan\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) \right] dx \\  &= \int \frac{\pi}{2} - \frac{\pi}{4} + \frac{x}{2} dx \\  &= \int \frac{\pi x}{4} + \frac{x^2}{2} dx \\  &= \int \frac{\pi}{4} + \frac{x^2}{2} + c \quad \text{ans.}  \end{aligned}  $
Q.4)	<p>(i) <math>I = \int \frac{\cos(2x)}{(\cos x + \sin x)^2} dx</math></p> <p>(ii) <math>I = \int \frac{\cos x - \sin x}{1 + \sin(2x)} dx</math></p> <p>(iii) <math>I = \int \frac{1 - \tan x}{1 + \tan x} dx</math></p>
Sol.4)	<p>(i) <math>I = \int \frac{\cos(2x)}{(\cos x + \sin x)^2} dx</math></p> $  \begin{aligned}  &= \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx \\  &= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2} dx \\  &= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx  \end{aligned}  $ <p>put <math>\cos x + \sin x = t</math></p> $  \begin{aligned}  &(-\sin x + \cos x) dx = dt \\  &= \int \frac{dt}{t} \\  &= \log  t  + c \\  &= I = \log  \cos x + \sin x  + c \quad \text{ans.}  \end{aligned}  $ <p>(ii) <math>I = \int \frac{\cos x - \sin x}{1 + \sin(2x)} dx</math></p> $  \begin{aligned}  &= \int \frac{\cos x - \sin x}{\sin^2 x + \cos^2 x + 2\sin x \cos x} dx \\  &= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx  \end{aligned}  $ <p>put <math>\sin x + \cos x = t</math></p> $  \begin{aligned}  &(\cos x - \sin x) dx = dt \\  &= \int \frac{dt}{t^2} \\  &= \int -\frac{1}{t} + c \\  &= I = -\frac{1}{\sin x + \cos x} + 2 \quad \text{ans.}  \end{aligned}  $ <p>(iii) <math>I = \int \frac{1 - \tan x}{1 + \tan x} dx</math></p> $  \begin{aligned}  &= \int \frac{\frac{1 - \sin x}{\cos x}}{\frac{1 + \sin x}{\cos x}} dx  \end{aligned}  $



	$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$ <p>put <math>\cos x + \sin x = t</math>  <math>(-\sin x + \cos x) dx = dt</math></p> $\therefore I = \int \frac{dt}{t}$ $= I = \log  \sin x + \cos x  + c \quad \text{ans.}$
Q.5)	(i) $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cdot \cos^2 x} dx$ (ii) $I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$
Sol.5)	(i) $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cdot \cos^2 x} dx$ $= \int \frac{(\sin^4 x)^2 - (\cos^4 x)^2}{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cdot \cos^2 x} dx \quad \dots \{1 = \sin^2 x + \cos^2 x\}$ $= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{\sin^4 x + \cos^4 x + 2\sin^2 x \cdot \cos^2 x - 2\sin^2 x \cdot \cos^2 x} dx$ $= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{\sin^4 x + \cos^4 x} dx$ $= \int (\sin^2 x + \cos x)(\sin^2 x - \cos^2 x) dx$ $= \int (1)[- \cos(2x)] dx \quad \dots \{\cos(2x) = \cos^2 x - \sin^2 x\}$ $= - \int \cos(2x) dx$ $= \frac{-\sin(2x)}{2} + c \quad \text{ans.}$  (ii) $I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$ $= \int \frac{1}{\sqrt{\sin^3 x \cdot (\sin x \cos \alpha + \cos x \sin \alpha)}} dx$ take $\sin x$ common $= \int \frac{1}{\sqrt{\sin^4 x \cdot (\cos \alpha + \cot x \sin \alpha)}} dx$ $= \int \frac{1}{\sqrt{\sin^2 x \cdot (\cos \alpha + \cot x \sin \alpha)}} dx$ $= \int \frac{\cosec^2 x}{\sqrt{\cos \alpha + \cot x \sin \alpha}} dx$ $= \int \frac{\cosec^2 x}{\sqrt{(\cos \alpha + \cot x \sin \alpha)}} dx$ put $\cos \alpha + \cot x \sin \alpha = t$ $\therefore -\cosec^2 x \cdot \sin x dx = dt$ $\cosec^2 x dx = \frac{-dt}{\sin x}$ $\therefore I = -\frac{1}{\sin x} \int \frac{dt}{\sqrt{t}}$ $= \frac{-1}{\sin x} \times 2\sqrt{t} + c$ $I = -\frac{1}{\sin x} 2\sqrt{\cos \alpha + \cot x \sin \alpha} + c \quad \text{ans.}$
Q.6)	(i) $I = \int \frac{\sin(2x)}{(a+b\cos x)^2} dx$ (ii) $I = \int \frac{1}{\sqrt{1-\sin x}} dx$
Sol.6)	(i) $I = \int \frac{\sin(2x)}{(a+b\cos x)^2} dx$



	$  \begin{aligned}  &= 2 \int \frac{\sin x \cos x}{(a+b \cos x)^2} dx \\  \text{put } a+b \cos x = t \\  \therefore -b \sin x dx = dt \\  \sin x dx &= \frac{-dt}{b} \\  I &= \frac{-2}{b} \int \frac{\cos x}{t^2} dt \\  &= \frac{-2}{b^2} \int \frac{1}{t^2} \cdot \left(\frac{t-a}{b}\right) dt \\  &= \frac{-2}{b^2} \int \frac{t-a}{t^2} dt \\  &= \frac{-2}{b^2} \int \frac{1}{t} - \frac{a}{t^2} dt \\  &= \frac{-2}{b^2} \left[ \log  t  + \frac{a}{t} \right] + c \\  I &= \frac{-2}{b^2} \left[ \log  a+b \cos x  + \frac{a}{a+b \cos x} \right] + c \quad \text{ans.}  \end{aligned}  $
	$  \begin{aligned}  \text{(ii)} I &= \int \frac{1}{\sqrt{1-\sin x}} dx \\  &= \int \frac{1}{\sqrt{1-\cos(\frac{\pi}{2}-x)}} dx \\  &= \int \frac{1}{\sqrt{2\sin^2(\frac{\pi}{4}-\frac{x}{2})}} dx \\  &= \frac{1}{\sqrt{2}} \int \operatorname{cosec} \left( \frac{\pi}{4} - \frac{x}{2} \right) dx \\  &= \frac{1}{\sqrt{2}} \cdot \log \left  \operatorname{cosec} \left( \frac{\pi}{4} - \frac{x}{2} \right) - \cot \left( \frac{\pi}{4} - \frac{x}{2} \right) \right  \times (-2) + c \\  &= -\sqrt{2} \cdot \log \left  \operatorname{cosec} \left( \frac{\pi}{4} - \frac{x}{2} \right) - \cot \left( \frac{\pi}{4} - \frac{x}{2} \right) \right  + c \quad \text{ans}  \end{aligned}  $
→	<p><b>Type :</b> <math>\int \text{linear} \sqrt{\text{Linear}} dx</math>, <math>\int \frac{\text{linear}}{\sqrt{\text{Linear}}} dx</math>, <math>\int \frac{\text{linear}}{\sqrt{(\text{Linear})^n}} dx</math></p> <p><b>Put Linear = t or <math>t^2</math> (Or) make adjustments</b></p>
Q.7)	<p>(i) <math>I = \int x \sqrt{x+2} dx</math>      (ii) <math>I = \int (7x-2) \sqrt{3x+2} dx</math></p> <p>(iii) <math>I = \int \frac{2x+3}{(x-1)^2} dx</math></p>
Sol.7)	<p>(i) <math>I = \int x \sqrt{x+2} dx</math>  put <math>x+2 = t^2</math>  <math>dx = 2tdt</math>  <math>\therefore I = 2 \int x \cdot \sqrt{t^2} tdt</math>  <math>= 2 \int (t^2 - 2) \cdot t \cdot tdt</math>  <math>= 2 \int t^4 - 2t^2 dt</math>  <math>= 2 \left[ \frac{t^5}{5} - \frac{2t^3}{3} \right] + c</math>  replacing t by <math>(x+2)^{\frac{1}{2}}</math></p>



$$= 2 \left[ \frac{(x+2)^{\frac{5}{2}}}{5} - 2 \frac{(x+2)^{\frac{3}{2}}}{3} \right] + c \quad \text{ans.}$$

Alternate Method: (adjustment)

$$\begin{aligned} I &= \int x\sqrt{x+2}dx \\ &= \int (x+2-2)\sqrt{x+2}dx \\ &= \int (x+2)^{\frac{3}{2}} - 2\sqrt{x+2}dx \\ &= \frac{2}{5}(x+2)^{\frac{5}{2}} - 2 \times \frac{2}{3}(x+2)^{\frac{3}{2}} + c \quad \text{ans.} \end{aligned}$$

$$(ii) I = \int (7x-2)\sqrt{3x+2}dx$$

$$\text{put } 3x+2 = t^2$$

$$3dx = 2tdt$$

$$dx = \frac{2}{3}tdt$$

$$\begin{aligned} I &= \frac{2}{3} \int (7x-2)\sqrt{t^2} \cdot tdt \\ &= \frac{2}{3} \int \left[ 7\left(\frac{t^2-2}{3}\right) - 2 \right] t \cdot tdt \\ &= \frac{2}{3} \int \frac{(7t^2-14-6)}{3} t^2 dt \\ &= \frac{2}{9} \int 7t^4 - 20t^2 dt \\ &= \frac{2}{9} \left[ \frac{7t^5}{5} - \frac{20t^3}{3} \right] + c \end{aligned}$$

replacing t by  $(3x+2)^{\frac{1}{2}}$

$$\therefore I = \frac{2}{9} \left[ \frac{7}{5}(3x+2)^{\frac{5}{2}} - \frac{20}{3}(3x+2)^{\frac{3}{2}} \right] + c \quad \text{ans.}$$

$$(iii) I = \int \frac{2x+3}{(x-1)^2} dx$$

$$\text{put } x-1 = t$$

$$dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{2x+3}{t^2} dt \\ &= \int \frac{2(t+1)+3}{t^2} dt \\ &= \int \frac{2t+5}{t^2} dt \\ &= \int \frac{2}{t} + \frac{5}{t^2} dt \\ &= 2\log|t| - \frac{5}{t} + c \\ I &= 2\log|x-1| - \frac{5}{x-1} + c \quad \text{ans.} \end{aligned}$$

Q.8)	(i) $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} dx$
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Sol.8)	(i) $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} dx$
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	Rationalize $\begin{aligned} I &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx \\ &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{a-b} dx \\ &= \frac{1}{a-b} \int \sqrt{x+a} - \sqrt{x+b} dx \\ &= \frac{1}{a-b} \left[ \frac{2}{3} (x+a)^{\frac{3}{2}} - \frac{2}{3} (x+b)^{\frac{3}{2}} \right] + c \end{aligned}$ <p style="text-align: right;">ans.</p>
Q.9)	(i) $I = \int \frac{(x^4-x)^{\frac{1}{4}}}{x^5} dx$ (ii) $I = \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx$
Sol.9)	(i) $I = \int \frac{(x^4-x)^{\frac{1}{4}}}{x^5} dx$ take $x^4$ common $\begin{aligned} &= \int \frac{x(1-\frac{x}{x^4})^{\frac{1}{4}}}{x^5} dx \\ &= \int \frac{(1-\frac{1}{x^3})^{\frac{1}{4}}}{x^4} dx \\ \text{put } 1 - \frac{1}{x^3} &= t \\ \frac{3}{x^4} dx &= dt \Rightarrow \frac{dx}{x^4} = \frac{dt}{3} \\ \therefore I &= \frac{1}{3} \int t^{\frac{1}{4}} dt \\ &= \frac{1}{3} \cdot \frac{4}{5} t^{\frac{5}{4}} + c \\ &= I = \frac{4}{15} \left( 1 - \frac{1}{x^3} \right)^{\frac{5}{4}} + c \end{aligned}$ <p style="text-align: right;">ans.</p> (ii) $I = \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx$ take $x^4$ common $\begin{aligned} &= \int \frac{1}{x^2 \cdot x^3 (1+\frac{1}{x^4})^{\frac{3}{4}}} dx \\ &= \int \frac{1}{x^5 (1+\frac{1}{x^4})^{\frac{3}{4}}} dx \\ \text{Put } \frac{1+1}{x^4} &= t \\ \frac{-4}{x^5} dx &= dt \Rightarrow \frac{dx}{x^5} = \frac{-dt}{5} \\ \therefore I &= \frac{-1}{5} \int \frac{1}{t^{\frac{3}{4}}} dt \\ &= \frac{-1}{5} \int t^{-\frac{3}{4}} dt \\ &= \frac{-1}{5} \left( t^{\frac{1}{4}} \times 4 \right) + c \\ &= \frac{-4}{5} \left( 1 + \frac{1}{x^4} \right)^{\frac{1}{4}} + c \end{aligned}$ <p style="text-align: right;">ans.</p>
Q.10)	(i) $I = \int \frac{1}{x\sqrt{ax-x^2}} dx$ (ii) $I = \int \frac{1}{x(x^n+1)} dx$

Sol.10) (i)  $I = \int \frac{1}{x\sqrt{ax-x^2}} dx$   
 take  $x^2$  common  
 $= \int \frac{1}{x \cdot x \sqrt{\frac{a}{x}-1}} dx$   
 $= \int \frac{1}{x^2 \sqrt{\frac{a}{x}-1}} dt$   
 put  $\frac{a}{x} - 1 = t$   
 $\frac{-a}{x^2} dx = dt \Rightarrow \frac{1}{x^2} dx = -\frac{dt}{a}$   
 $I = \frac{-1}{a} \int \frac{dt}{\sqrt{t}}$   
 $= \frac{-1}{a} \times 2\sqrt{t} + c$   
 $= I = \frac{-2}{a} \sqrt{\frac{a}{x}-1} + c$  ans.

(ii)  $I = \int \frac{1}{x(x^{n+1})} dx$   
 take  $x^n$  common  
 $= \int \frac{1}{x^{n+1}(1+\frac{1}{x^n})} dx$   
 put  $\frac{1+1}{x^n} = t$   
 $\frac{-n}{x^{n+1}} dx = dt$   
 $\Rightarrow \frac{1}{x^{n+1}} dx = -\frac{dt}{n}$   
 $\therefore I = \frac{-1}{n} \int \frac{1}{t} dt$   
 $= \frac{-1}{n} \log |t| + c$   
 $= \frac{-1}{n} \log |1 + \frac{1}{x^n}| + c$  ans.