



Integration (Indefinite Integrals)

→	Type: Integration By Parts :	
Q.1)	(a) $I = \int x^2 \sin x \, dx$	(b) $I = \int x \sin^2 x \, dx$
Sol.1)	$ \begin{aligned} (a) I &= \int x^2 \sin x \, dx \\ &= x^2(-\cos x) - \int 2x.(-\cos x) \, dx \\ &= -x^2 \cos x - 2 \int x \cos x \, dx \\ &= -x^2 \cos x - 2[x(\sin x) - \int (1). \sin x \, dx] \\ &= -x^2 \cos x - 2[x \sin x + \cos x] + c \quad \text{ans.} \end{aligned} $ $ \begin{aligned} (b) I &= \int x \sin^2 x \, dx \\ &= \int x \left(\frac{1-\cos(2x)}{2} \right) \, dx \\ &= \frac{1}{2} \int x - x \cos(2x) \, dx \\ &= \frac{1}{2} \int x \, dx - \frac{1}{2} \int x \cos(2x) \, dx \\ &= \frac{1}{2} \cdot \frac{x^2}{2} - \frac{1}{2} \left[x \frac{\sin(2x)}{2} - \int (1) \cdot \frac{\sin(2x)}{2} \, dx \right] \\ &= \frac{x^2}{4} - \frac{1}{2} \left[\frac{1}{2} x \sin(2x) - \frac{1}{2} \int \sin(2x) \, dx \right] \\ &= I = \frac{x^2}{4} - \frac{1}{2} \left[\frac{1}{2} x \sin(2x) + \frac{1}{4} \cos(2x) \right] + c \quad \text{ans.} \end{aligned} $	
Q.2)	$ \begin{aligned} (a) I &= \int \log x \, dx \\ (c) I &= \int \frac{\log x}{x^2} \, dx \end{aligned} $	$(b) I = \int (\log x)^2 \, dx$
Sol.2)	$ \begin{aligned} (a) I &= \int \log x \, dx \\ &= \int \log x \cdot 1 \, dx \\ &= \log x \cdot (x) - \int \frac{1}{x} \cdot (x) \, dx \\ &= x \log x - \int 1 \, dx \\ I &= x \log x - x + c \quad \text{ans.} \end{aligned} $ $ \begin{aligned} (b) I &= \int (\log x)^2 \, dx \\ &= \int (\log x)^2 \cdot 1 \, dx \\ &= (\log x)^2 \cdot x - \int \frac{2 \log x}{x} \cdot x \, dx \\ &= x(\log x)^2 - 2 \int \log x \cdot 1 \, dx \\ &= x(\log x)^2 - 2 \left[\log x \cdot (x) - \int \frac{1}{x} \cdot x \, dx \right] \\ &= x(\log x)^2 - 2[x \log x - x] + c \quad \text{ans.} \end{aligned} $ $ \begin{aligned} (c) I &= \int \frac{\log x}{x^2} \, dx \\ &= \int \frac{1}{x^2} \cdot \log x \, dx \\ &= \log x \left(\frac{-1}{x} \right) - \int \frac{1}{x} \left(\frac{-1}{x} \right) \, dx \\ &= \frac{-1}{x} \log x + \int \frac{1}{x^2} \, dx \end{aligned} $	

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	$= \frac{-1}{x} \log x - \frac{1}{x} + c$ ans.
Q.3)	(a) $I = \int \sin^{-1} x \, dx$ (b) $I = \int (\sin^{-1} x)^2 \, dx$
Sol.3)	$ \begin{aligned} (a) I &= \int \sin^{-1} x \, dx \\ &= \int \sin^{-1} x \cdot 1 \, dx \\ &= \sin^{-1}(x) - \int \frac{1}{\sqrt{1-x^2}} \cdot x \, dx \\ \text{put } 1-x^2 &= t \\ -2x \, dx &= dt \\ x \, dx &= -\frac{dt}{2} \\ \therefore I &= x \sin^{-1} x + \frac{1}{2} \int \frac{dt}{\sqrt{t}} \\ &= x \sin^{-1} x + \frac{1}{2} \times 2\sqrt{t} + c \\ I &= x \sin^{-1} x + \sqrt{1-x^2} + c \quad \text{ans.} \end{aligned} $ $ \begin{aligned} (b) I &= \int (\sin^{-1} x)^2 \, dx \\ &= \int (\sin^{-1} x)^2 \cdot 1 \, dx \\ &= (\sin^{-1} x)^2 \cdot x - \int 2 \frac{\sin^{-1} x}{\sqrt{1-x^2}} \cdot x \, dx \\ &= x(\sin^{-1} x)^2 - 2 \int x \frac{\sin^{-1} x}{\sqrt{1-x^2}} \, dx \\ \text{put } \sin^{-1} x &= t \quad (x = \sin t) \\ \frac{1}{\sqrt{1-x^2}} \, dx &= dt \\ \therefore I &= x(\sin^{-1} x)^2 - 2 \int \sin t \cdot t \, dt \\ &= x(\sin^{-1} x)^2 - 2[t(-\cos t) - \int (1)(-\cos t) \, dt] \\ &= x(\sin^{-1} x)^2 - 2[-t\cos t + \sin t] + c \\ &= x(\sin^{-1} x)^2 - 2[-\sin^{-1} x \sqrt{1-x^2} + x] + c \quad \text{ans.} \\ &\quad \left\{ \sin t = x \text{ then } \cos t = \sqrt{1-\sin^2 t} = \sqrt{1-x^2} \right\} \end{aligned} $
Q.4)	(a) $I = \int \frac{x \tan^{-1} x^{\frac{3}{2}}}{(1+x^2)^2} \, dx$ (b) $I = x \frac{x^2 \sin^{-1} x^{\frac{3}{2}}}{(1-x^2)^2} \, dx$
Sol.4)	$ \begin{aligned} (a) I &= \int \frac{x \tan^{-1} x^{\frac{3}{2}}}{(1+x^2)^2} \, dx \\ &= \int \frac{x \tan^{-1} x}{\sqrt{1+x^2}(1+x^2)} \, dx \\ \text{put } \tan^{-1} x &= t \quad \therefore \frac{1}{1+x^2} \, dx = dt \\ \text{also } x &= \tan t \\ \therefore I &= \int \frac{xt}{\sqrt{1+x^2}} \, dt \\ \text{put } x &= \tan t \\ I &= \int \frac{\tan t \cdot t}{\sqrt{1+\tan^2 t}} \, dt \\ &= \int t \frac{\tan t}{\sec t} \, dt \\ &= \int t \sin t \, dt \\ &= t(-\cos t) - \int (1)(-\cos t) \, dt \end{aligned} $



	$= -t \cos t + \sin t + c$ $\left[\begin{array}{l} \therefore \tan t = x \\ P = x, B = 1 \\ H = \sqrt{x^2 + 1} \\ \cos t = \frac{B}{H}, \sin t = \frac{B}{H} \end{array} \right]$ $= -\tan^{-1}x \cdot \frac{1}{\sqrt{x^2+1}} + \frac{x}{\sqrt{1+x^2}} + c \quad \text{ans.}$ $(b) I = \int \frac{x^2 \sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} dx$ $= \int \frac{x^2 \sin^{-1}x}{(1-x^2)\sqrt{1-x^2}} dx$ <p>put $\sin^{-1}x = t \quad (\sin t = x)$</p> $\therefore \frac{1}{\sqrt{1-x^2}} dx = dt$ $\therefore I = \int \frac{x^2 t}{(1-x^2)} dt$ <p>put $\sin t = x$</p> $I = \int \frac{\sin^2 t \cdot t}{1-\sin^2 t} dt$ $= \int \tan^2 t \cdot t dt$ $= \int (\sec^2 t - 1) t dt$ $= \int t \sec^2 t - t dt$ $= \int t \sec^2 t dt - \int t dt$ $\left[\begin{array}{l} \therefore \sin t = x \\ P = x, H = 1 \\ B = \sqrt{1-x^2} \\ \tan t = \frac{P}{B} \\ \sec t = \frac{H}{B} \end{array} \right]$ $= t(\tan t) - \int (1) \cdot \tan t dt - \frac{t^2}{2}$ $= t \tan t - \log \sec t - \frac{t^2}{2} + c$ $= \sin^{-1}x \cdot \frac{x}{\sqrt{1-x^2}} - \log \left \frac{1}{\sqrt{1-x^2}} \right - \frac{(\sin^{-1}x)^2}{2} + c$ $= \frac{x \sin^{-1}x}{\sqrt{1-x^2}} + \frac{1}{2} \log 1-x^2 - \frac{(\sin^{-1}x)^2}{2} + c \quad \text{ans.}$
Q.5)	$I = \int \frac{x-\sin x}{1-\cos x} dx$
Sol.5)	$(a) I = \int \frac{x-\sin x}{1-\cos x} dx$ $= \int \frac{x-2\sin^2 \frac{x}{2} \cdot \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}} dx$ <p>Separate</p> $= \frac{1}{2} \int x \operatorname{cosec}^2 \left(\frac{x}{2} \right) dx - \int \cot \left(\frac{x}{2} \right) dx$ $= \frac{1}{2} \left[x \left(-2 \cot \left(\frac{x}{2} \right) \right) - \int 2 \left(-\cot \left(\frac{x}{2} \right) dx \right) \right] - 2 \log \left \sin \left(\frac{x}{2} \right) \right $ $= \frac{1}{2} \left[-2x \cot \left(\frac{x}{2} \right) + 2 \int \cot \left(\frac{x}{2} \right) dx \right] - 2 \log \left \sin \frac{x}{2} \right $ $= \frac{1}{2} \left[-2x \cot \left(\frac{x}{2} \right) + 2 \times 2 \log \left \sin \left(\frac{x}{2} \right) \right \right] - 2 \log \left \sin \frac{x}{2} \right $



	$= -x \cot\left(\frac{x}{2}\right) + 2 \log \left \sin\left(\frac{x}{2}\right) \right - 2 \log \left \sin\left(\frac{x}{2}\right) \right + c$ $= -x \cot\left(\frac{x}{2}\right) + c \quad \text{ans.}$
Q.6)	(a) $I = \int x \sin^{-1} x \, dx$ (b) $I = \int \sin^{-1} \sqrt{x} \, dx$
Sol.6)	$(a) I = \int x \sin^{-1} x \, dx$ $= \sin^{-1} x \cdot \frac{x^2}{2} - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} \, dx$ $= \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} \, dx$ $= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} \, dx$ $= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} \, dx$ $= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \, dx$ $= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1}(x) - \sin^{-1}(x) \right] + c$ $\dots \left[\dots \int \sqrt{a^2-x^2} = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) \right]$ $= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \left[\frac{x}{2} \sqrt{1-x^2} - \frac{1}{2} \sin^{-1} x \right] + c \quad \text{ans.}$ $(b) I = \int \sin^{-1} \sqrt{x} \, dx$ <p>put $\sqrt{x} = t$ $x = t^2$ $dx = 2t \, dt$ $\therefore I = 2 \int \sin^{-1} t \cdot t \, dt$ Proceed as above Ques.</p> $x \sin^{-1} x + \frac{1}{2} \sqrt{x-x^2} - \frac{1}{2} \sin^{-1} \sqrt{x} + c \quad \text{ans.}$
Q.7)	(a) $I = \int \frac{\sin^{-1} x}{x^2} \, dx$ (b) $I = \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} \, dx$
Sol.7)	$(a) I = \int \frac{\sin^{-1} x}{x^2} \, dx$ <p>put $\sin^{-1} x = t$ $x = \sin t$ $\therefore dx = \cos t \, dt$ $\therefore I = \int \frac{t \cos t}{\sin^2 t} \, dt$ $= \int t \operatorname{cosec} t \cdot \cot t \, dt$ $= t(-\operatorname{cosec} t) - \int (1)(-\operatorname{cosec} t) dt$ $= \frac{-t}{\sin t} + \int \operatorname{cosec} t \, dt$ $= \frac{-t}{\sin t} + \log \operatorname{cosec} t - \cot t + c$ $= \frac{-\sin^{-1} x}{x} + \log \left \frac{1}{\sin t} - \frac{\cot t}{\sin t} \right + c$ $= \frac{-\sin^{-1} x}{x} + \log \left \frac{1}{x} - \frac{\sqrt{1-x^2}}{x} \right + c \quad \text{ans.}$ $(b) I = \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} \, dx$ </p>

Property : $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$. . . $\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$

$$I = \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\frac{\pi}{2}} dx$$

$$= \frac{2}{\pi} \int 2\sin^{-1}\sqrt{x} - \frac{\pi}{2} dx$$

$$= \frac{4}{\pi} \int \sin^{-1}\sqrt{x} dx - \int 1 dx$$

put $\sqrt{x} = t$

$$x = t^2$$

$$dx = 2t dt$$

$$\therefore I = \frac{8}{\pi} \int \sin^{-1}t \cdot t dt - x$$

$$= \frac{8}{\pi} \left[\sin^{-1}t \cdot \frac{t^2}{2} - \frac{1}{2} \int \frac{1}{\sqrt{1-t^2}} \cdot t^2 dt \right] - x$$

$$= \frac{8}{\pi} \left[\frac{t^2}{2} \cdot \sin^{-1}t + \frac{1}{2} \int \frac{-t^2}{\sqrt{1-t^2}} dt \right] - x$$

$$= \frac{8}{\pi} \left[\frac{t^2}{2} \sin^{-1}t + \frac{1}{2} \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt \right] - x$$

$$= \frac{8}{\pi} \left[\frac{t^2}{2} \sin^{-1}t + \frac{1}{2} \int \sqrt{1-t^2} - \frac{1}{\sqrt{1-t^2}} dt \right] - x$$

$$= \frac{8}{\pi} \left[\frac{t^2}{2} \sin^{-1}t + \frac{1}{2} \left\{ \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1}(t) - \sin^{-1}(t) \right\} \right] - x + c$$

$$= \frac{8}{\pi} \left[\frac{t^2}{2} \sin^{-1}t + \frac{1}{2} \left\{ \frac{t}{2} \sqrt{1-t^2} - \frac{1}{2} \sin^{-1}t \right\} \right] - x + c$$

$$= \frac{8}{\pi} \left[\frac{t^2}{2} \sin^{-1}t + \frac{t}{4} \sqrt{1+t^2} - \frac{1}{4} \sin^{-1}t \right] - x + c$$

replace t by \sqrt{x}

$$= \frac{8}{\pi} \left[\frac{x}{2} \sin^{-1}\sqrt{x} + \frac{\sqrt{x}\sqrt{1-x}}{4} - \frac{\sin^{-1}\sqrt{x}}{4} \right] - x + c \text{ ans.}$$

Q.8)

$$(a) I = \int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4} dx$$

$$(b) I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$$

Sol.8)

$$(a) I = \int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4} dx$$

$$= \int \frac{\sqrt{x^2+1}(\log(x^2+1)-\log(x^2))}{x^4} dx$$

$$= \int \frac{\sqrt{x^2+1} \left[\log \frac{(x^2+1)}{x^2} \right]}{x^4} dx$$

take x^2 common

$$= \int \frac{x \sqrt{1+\frac{1}{x^2}} \cdot \log \left(1 + \frac{1}{x^2} \right)}{x^4} dx$$

$$= \int \frac{\sqrt{1+\frac{1}{x^2}} \cdot \log \left(1 + \frac{1}{x^2} \right)}{x^3} dx$$

put $1 + \frac{1}{x^2} = t$

$$\frac{-2}{x^3} dx = dt \quad \frac{1}{x^3} dx = \frac{-dt}{2}$$

$$\therefore I = \frac{-1}{2} \int \sqrt{t} \cdot \log t dt$$

$$= \frac{-1}{2} \left[\log t \cdot \frac{2}{3} (t)^{\frac{3}{2}} - \frac{2}{3} \int \frac{1}{t} \cdot t^{\frac{3}{2}} dt \right]$$



	$ \begin{aligned} &= \frac{-1}{2} \left[\frac{2}{3} \log t \cdot t^{\frac{3}{2}} - \frac{2}{3} \int t^{\frac{1}{2}} dt \right] \\ &= \frac{-1}{2} \left[\frac{2}{3} \log t \cdot t^{\frac{3}{2}} - 4t^{\frac{3}{2}} \right] + c \\ &= \frac{-1}{2} \times \frac{2}{3} \times t^{\frac{3}{2}} \left[\log t - \frac{2}{3} \right] + c \end{aligned} $ <p>replacing t</p> $= \frac{-1}{3} \left(1 + \frac{1}{x^2} \right)^{\frac{3}{2}} \cdot \left[\log \left(1 + \frac{1}{x^2} \right) - \frac{2}{3} \right] + c \text{ ans.}$
	<p>(b) $I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$</p> <p>put $x = \cos(2\theta)$</p> $ \begin{aligned} \therefore I &= \int \tan^{-1} \sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}} dx \\ &= \int \tan^{-1} \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} dx \\ &= \int \tan^{-1}(\tan\theta) dx \\ &= \int \theta \cdot dx \end{aligned} $ <p>replacing θ</p> $ \begin{aligned} &= \frac{1}{2} \int \cos^{-1} x dx \\ &= \frac{1}{2} \int \cos^{-1} x \cdot 1 dx \\ &= \frac{1}{2} \left[\cos^{-1} x \cdot x - \int \frac{-1}{\sqrt{1-x^2}} \cdot x dx \right] \\ &= \frac{1}{2} \left[x \cos^{-1} x + \int \frac{x}{\sqrt{1-x^2}} dx \right] \end{aligned} $ <p>put $1-x^2 = t \quad \therefore xdx = \frac{-dt}{2}$</p> $ \begin{aligned} \therefore I &= \frac{1}{2} \left[x \cos^{-1} x - \frac{1}{2} \int \frac{dt}{\sqrt{t}} \right] \\ &= \frac{1}{2} \left[x \cos^{-1} x - \frac{1}{2} \times 2\sqrt{t} \right] + c \\ &= \frac{1}{2} \left[x \cos^{-1} x - \sqrt{1-x^2} \right] + c \text{ ans.} \end{aligned} $
Q.9)	<p>(a) $I = \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$</p> <p>(b) $I = \int \frac{x^2}{(x \sin x + \cos x)^2} dx$</p>
Sol.9)	<p>(a) $I = \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$</p> <p>put $x = a \tan^2 \theta$</p> $dx = 2a \tan \theta \cdot \sec^2 \theta d\theta$ $ \begin{aligned} \therefore I &= 2a \int \sin^{-1} \sqrt{\frac{a \tan^2 \theta}{a+a \tan^2 \theta}} \cdot \tan \theta \cdot \sec^2 \theta d\theta \\ &= 2a \int \sin^{-1} \sqrt{\frac{\tan^2 \theta}{\sec^2 \theta}} \cdot \tan \theta \cdot \sec^2 \theta d\theta \\ &= 2a \int \sin^{-1} \sqrt{\sin^2 \theta} \cdot \tan \theta \cdot \sec^2 \theta d\theta \\ &= 2a \int \sin^{-1}(\sin \theta) \cdot \tan \theta \cdot \sec^2 \theta d\theta \\ &= 2a \int \theta \cdot \tan \theta \cdot \sec^2 \theta d\theta \\ &= 2a \left[\theta \cdot \int \tan \theta \sec^2 \theta d\theta - \int (1 \cdot \int \tan \theta \sec^2 \theta d\theta) d\theta \right] \end{aligned} $



put $\tan\theta = t$ in both integrals

$$\begin{aligned}\therefore \sec^2\theta d\theta &= dt \\ &= 2a[\theta \int t dt - \int (\int t dt) d\theta]\end{aligned}$$

$$= 2a\left[\theta \cdot \frac{t^2}{2} - \int \left(\frac{t^2}{2}\right) d\theta\right]$$

replacing t

$$\begin{aligned}&= 2a\left[\theta \cdot \frac{\tan^2\theta}{2} - \frac{1}{2} \int \tan^2\theta d\theta\right] \\ &= 2a\left[\theta \cdot \frac{\tan^2\theta}{2} - \frac{1}{2} \int (\sec^2\theta - 1) d\theta\right] \\ &= 2a\left[\theta \cdot \frac{\tan^2\theta}{2} - \frac{1}{2} \{\tan\theta - \theta\}\right] + c\end{aligned}$$

replacing θ by $\tan^{-1} \sqrt{\frac{x}{a}}$

$$\begin{aligned}&= a\left[\tan^{-1} \sqrt{\frac{x}{a}} \cdot \frac{x}{a} - \left\{\sqrt{\frac{x}{a}} - \tan^{-1} \sqrt{\frac{x}{a}}\right\}\right] + c \\ &= a\left[\frac{x}{a} \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1} \sqrt{\frac{x}{a}}\right] + c \\ &= a\left[\tan^{-1} \sqrt{\frac{x}{a}} \left(\frac{x}{a} + 1\right) - \sqrt{\frac{x}{a}}\right] + c \text{ ans.}\end{aligned}$$

$$(b) I = \int \frac{x^2}{(x \sin x + \cos x)^2} dx$$

adjustment

$$\begin{aligned}&= \int \frac{x \cdot x \cdot \cos x \cdot \sec x}{(x \sin x + \cos x)^2} dx \\ &= (x \sec x) \cdot \int \frac{x \cos x}{(x \sin x + \cos x)^2} dx\end{aligned}$$

Both parts

$$= x \sec x \cdot \int \frac{x \cos x}{(x \sin x + \cos x)^2} - \int \frac{d}{dx}(x \sec x) \cdot \int \frac{x \cos x}{(x \sin x + \cos x)^2} dx$$

put $x \sin x + \cos x = t$ in both integrals

$$(x \cos x + \sin x - \sin) dx = dt$$

$$x \cos x dx = dt$$

$$\therefore I = x \sec x \int \frac{dt}{t^2} - \int (x \sec x \tan x + \sec x) \cdot \int \frac{dt}{t^2}$$

$$= x \sec x \left[\frac{-1}{t} \right] - \int (x \sec x \tan x + \sec x) \left(\frac{-1}{t} \right) dx$$

replacing t by $x \sin x + \cos x = t$

$$\begin{aligned}&= \frac{-x \sec x}{x \sin x + \cos x} - \int (x \sec x \tan x + \sec x) \left(\frac{-1}{x \sin x + \cos x} \right) dx \\ &= \frac{-x \sec x}{x \sin x + \cos x} + \int \sec x (x \tan x + 1) \frac{1}{x \sin x + \cos x} dx \\ &= \frac{-x \sec x}{x \sin x + \cos x} + \int \sec x \left(\frac{x \sin x + \cos x}{\cos x} \right) \left(\frac{1}{x \sin x + \cos x} \right) dx \\ &= \frac{-x \sec x}{x \sin x + \cos x} - \int \sec^2 x dx \\ &= \frac{-x \sec x}{x \sin x + \cos x} + \tan x + c \text{ ans.}\end{aligned}$$

Q.10) $I = \int \sec^3 x dx$

Sol.10) $I = \int \sec^3 x dx$
 $= \int \sec x \cdot \sec^2 x dx$

	$ \begin{aligned} &= \sec x \cdot \tan x - \int \sec x \cdot \tan x \cdot \tan x dx \\ &= \sec x \cdot \tan x - \int \sec x (\sec^2 x - 1) dx \\ &= \sec x \cdot \tan x - \int \sec^3 x - \sec x dx \\ &= \sec x \cdot \tan x - \int \sec^3 x dx + \int \sec x dx \\ I &= \sec x \cdot \tan x - I + \log \sec x + \tan x \\ 2I &= \sec x \tan x + \log \sec x + \tan x + c \\ \therefore I &= \frac{1}{2} [\sec x \tan x + \log \sec x + \tan x] + c \text{ ans.} \end{aligned} $
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