

TOPIC 6
INDEFINITE & DEFINITE INTEGRALS
SCHEMATIC DIAGRAM

Topics	Concept	Degree of Importance	References
			Text book of NCERT, Vol. II 2007 Edition
Indefinite Integrals	(i) Integration by substitution (ii) Application of trigonometric function in integrals (iii) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px+q)dx}{ax^2 + bx + c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	*	Exp 5&6 Page301,303
	(iv) Integration using Partial Fraction (v) Integration by Parts (vi) Some Special Integrals $\int \sqrt{a^2 \pm x^2} dx$, $\int \sqrt{x^2 - a^2} dx$	**	Ex 7 Page 306, Exercise 7.4Q13&Q24 Exp 8, 9, 10 Page 311,312 Exercise 7.4 Q 3,4,8,9,13&23 Exp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
	(vii) Miscellaneous Questions	***	Exp 18,19&20 Page 325 Exp 23 &24 Page 329 Solved Ex.41
Definite Integrals	(i) Definite Integrals based upon types of indefinite integrals (ii) Definite integrals as a limit of sum (iii) Properties of definite Integrals (iv) Integration of modulus function	* ** *** **	Exercise 27 Page 336, Q 2,3,4,5,9,11,16 Exercise 7.9 Exp 25 &26 Page 333, 334 Q3, Q5 & Q6 Exercise 7.8 Exp 31 Page 343*,Exp 32*,34&35 page 344 Exp 36***Exp 346 Exp 44 page351 Exercise 7.11 Q17 & 21 Exp 30 Page 343,Exp 43 Page 351 Q5& Q6 Exercise 7.11

SOME IMPORTANT RESULTS/CONCEPTS

$* \int x^n dx = \frac{x^{n+1}}{n+1} + C$ $* \int 1 dx = x + C$ $* \int \frac{1}{x^n} dx = -\frac{1}{x^{n-1}} + C$ $* \int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$ $* \int \frac{1}{x} dx = \ln x + C$ $* \int e^x dx = e^x + C$ $* \int a^x dx = \frac{a^x}{\ln a} + C$ $* \int \sin x dx = -\cos x + C$ $* \int \cos x dx = -\sin x + C$ $* \int \sec^2 x dx = \tan x + C$ $* \int \csc^2 x dx = -\cot x + C$ $* \int \sec x \cdot \tan x dx = \sec x + C$ $* \int \csc x \cdot \cot x dx = -\csc x + C$ $* \int \tan x dx = -\ln \cos x + C = \ln \sec x + C$ $* \int \cot x dx = \ln \sin x + C$ $* \int \sec x dx = \ln \sec x + \tan x + C$ $\quad \quad \quad = \ln\left \tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right + C$ $* \int \cosec x dx = \ln \cosec x - \cot x + C$ $\quad \quad \quad = -\ln \cosec x + \cot x + C$ $= \ln\left \tan\frac{x}{2}\right + C$ $* \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left \frac{x-a}{x+a}\right + C, \text{ if } x > a$ $* \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left \frac{a+x}{a-x}\right + C, \text{ if } x > a$ $* \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left \frac{a+x}{a-x}\right + C, \text{ if } x > a$	$* \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C = -\cos^{-1} \frac{x}{a} + C$ $* \int \frac{dx}{\sqrt{a^2 + x^2}} = \ln x + \sqrt{x^2 + a^2} + C$ $* \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln x + \sqrt{x^2 - a^2} + C$ $*$ $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln x + \sqrt{x^2 + a^2} + C$ $*$ $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln x + \sqrt{x^2 - a^2} + C$ $* \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$ $* \int \{f_1(x) \pm f_2(x) \pm \dots \pm f_n(x)\} dx$ $\quad \quad \quad = \int f_1(x) dx \pm \int f_2(x) dx \pm \dots \pm \int f_n(x) dx$ $* \int \lambda f(x) dx = \lambda \int f(x) dx + C$ $* \int u.v dx = u \int v dx - \int \left[\int v dx \right] \frac{du}{dx} dx$ $* \int_a^b f(x) dx = F(b) - F(a), \text{ where } F(x) = \int f(x) dx$ * General Properties of Definite Integrals. $* \int_a^b f(x) dx = \int_a^b f(t) dt$ $* \int_a^b f(x) dx = - \int_a^b f(x) dx$ $* \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ $* \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$ $* \int_0^a f(x) dx = \int_0^a f(a-x) dx$ $* \int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$
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$*\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, = -\frac{1}{a} \cot^{-1} \frac{x}{a} + C$	$*\int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a - x) = f(x). \\ 0 & \text{if } f(2a - x) = -f(x) \end{cases}$
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Assignments

(i) *Integration by substitution*

LEVEL I

1. $\int \frac{\sec^2(\log x)}{x} dx$

2. $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$

3. $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$

LEVEL II

1. $\int \frac{1}{\sqrt{x+x}} dx$

2. $\int \frac{1}{x\sqrt{x^6-1}} dx$

3. $\int \frac{1}{e^x - 1} dx$

LEVEL III

1. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

2. $\int \frac{\tan x}{\sec x + \cos x} dx$

3. $\int \frac{1}{\sin x \cos^3 x} dx$

(ii) *Application of trigonometric function in integrals*

LEVEL I

1. $\int \sin^3 x dx$

2. $\int \cos^2 3x dx$

3. $\int \cos x \cos 2x \cos 3x dx$

LEVEL II

1. $\int \sec^4 x \tan x dx$

2. $\int \frac{\sin 4x}{\sin x} dx$

LEVEL III

1. $\int \cos^5 x dx$

2. $\int \sin^2 x \cos^3 x dx$

(iii) *Integration using standard results*

LEVEL I

1. $\int \frac{dx}{\sqrt{4x^2 - 9}}$

2. $\int \frac{1}{x^2 + 2x + 10} dx$

3. $\int \frac{dx}{9x^2 + 12x + 13}$

LEVEL II

1. $\int \frac{x}{x^4 + x^2 + 1} dx$

2. $\int \frac{\cos x}{\sin^2 x + 4 \sin x + 5} dx$

3. $\int \frac{dx}{\sqrt{7 - 6x - x^2}}$

LEVEL III

1. $\int \frac{2x}{\sqrt{1-x^2-x^4}} dx$

2. $\int \frac{x^2+x+1}{x^2-x+1} dx$

3. $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$

4. $\int \sqrt{\frac{1-x}{1+x}} dx$

5. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} [CBSE 2011]$

(iv) Integration using Partial Fraction

LEVEL I

1. $\int \frac{2x+1}{(x+1)(x-1)} dx$

2. $\int \frac{x^2}{(x-1)(x-2)(x-3)} dx$

3. $\int \frac{3x-2}{(x+1)^2(x+3)} dx$

LEVEL II

1. $\int \frac{x^2+2x+8}{(x-1)(x-2)} dx$

2. $\int \frac{x^2+x+1}{x^2(x+2)} dx$

3. $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$

LEVEL III

1. $\int \frac{8}{(x+2)(x^2+4)} dx$

2. $\int \frac{dx}{\sin x + \sin 2x}$

3. $\int \frac{1}{1+x^3} dx$

(v) Integration by Parts

LEVEL I

1. $\int x \cdot \sec^2 x dx$

2. $\int \log x dx$

3. $\int e^x (\tan x + \log \sec x) dx$

LEVEL II

1. $\int \sin^{-1} x dx$

2. $\int x^2 \cdot \sin^{-1} x dx$

3. $\int \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}} dx$

4. $\int \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$

5. $\int \sec^3 x dx$

LEVEL III

1. $\int \cos(\log x) dx$

2. $\int \frac{e^x(1+x)}{(2+x)^2} dx$

3. $\int \frac{\log x}{(1+\log x)^2} dx$

4. $\int \frac{2+\sin x}{1+\cos 2x} e^x dx$

5. $\int e^{2x} \cdot \cos 3x dx$

(vi) Some Special Integrals

LEVEL I

1. $\int \sqrt{4+x^2} dx$

2. $\int \sqrt{1-4x^2} dx$

LEVEL II

1. $\int \sqrt{x^2+4x+6} dx$

2. $\int \sqrt{1-4x-x^2} dx$

LEVEL III

1. $\int (x+1)\sqrt{1-x-x^2} dx$ 2. $\int (x-5)\sqrt{x^2+x} dx$

(vii) Miscellaneous Questions

LEVEL II

1. $\int \frac{1}{2-3\cos 2x} dx$

2. $\int \frac{1}{3+\sin 2x} dx$

3. $\int \frac{dx}{4\sin^2 x + 5\cos^2 x}$

4. $\int \frac{dx}{1+3\sin^2 x + 8\cos^2 x}$

5. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$ 6. $\int \frac{\sec x}{5\sec x + 4\tan x} dx$

LEVEL III

1. $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx$

2. $\int \frac{dx}{1-\tan x}$

3. $\int \frac{x^4}{x^4 - 1} dx$

4. $\int \frac{x^2 + 1}{x^4 + x^2 + 1} dx$

5. $\int \frac{x^2 - 1}{x^4 + 1} dx$

6. $\int \sqrt{\tan x} dx$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I

1. $\int_0^1 \frac{2x+3}{5x^2+1} dx$

2. $\int_0^{\pi/2} \sqrt{\sin x} \cdot \cos^5 x dx$

3. $\int_0^2 x \sqrt{x+2} dx$

LEVEL II

1. $\int_1^2 \frac{5x^2}{x^2 + 4x + 3} dx$

2. $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$

(ii) Definite integrals as a limit of sum

LEVEL I

1. Evaluate $\int_0^2 (x+2) dx$ as the limit of a sum.

2. Evaluate $\int_0^4 (1+x) dx$ definite integral as the limit of a sum.

LEVEL II

1. Evaluate $\int_1^2 (3x^2 - 1) dx$ as the limit of a sum.

2. Evaluate $\int_0^3 (x^2 + 1) dx$ as the limit of a sum.

LEVEL III

1. Evaluate $\int_1^2 (x^2 + x + 2) dx$ as the limit of a sum.

2. Evaluate $\int_2^4 (e^{2x} + x^2) dx$ as the limit of a sum.

(iii) Properties of definite Integrals

LEVEL I

1. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx$

2. $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}} dx$

3. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

LEVEL II

1. $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$

2. $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

3. $\int_0^{\pi} \frac{x \tan x}{\sec x \cdot \csc x} dx$

4. $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ [CBSE 2011]

LEVEL III

1. $\int_0^{\pi} \frac{x + \sin x}{1 + \cos x} dx$ [CBSE 2011] 2. $\int_0^{\pi/2} \log \sin x dx$

3. $\int_0^{\pi/4} \log(1 + \tan x) dx$

[CBSE 2011]

(iv) Integration of modulus function

LEVEL III

1. $\int_2^5 (|x-2| + |x-3| + |x-4|) dx$

2. $\int_{-1}^2 |x^3 - x| dx$

3. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|] dx$

Questions for self evaluation

1. Evaluate $\int \frac{(2x-3)dx}{x^2 - 3x - 18}$

2. Evaluate $\int \frac{(3x+1).dx}{\sqrt{5-2x-x^2}}$

3. Evaluate $\int \cos^4 x dx$

5. Evaluate $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

7. Evaluate $\int_0^{\pi/2} \sqrt{\sin x} \cdot \cos^5 x dx$

9. Evaluate $\int_0^{\pi/2} \log \sin x dx$

4. Evaluate $\int \frac{dx}{3+2\sin x + \cos x}$

6. Evaluate $\int \frac{x \cdot \sin^{-1} x}{\sqrt{1-x^2}} dx$

8. Evaluate $\int_{-1}^{3/2} |x \sin \pi x| dx$

10. Evaluate $\int_1^4 (|x-1| + |x-2| + |x-3|) dx$