

## Indefinite Integrals

### Key points to remember

- **Integration is the reverse process of differentiation.**
- Let  $\frac{d}{dx} F(x) = f(x)$  then write  $\int f(x)dx = F(x) + C$ .
- These integrals are called **indefinite integrals** and c is called **constant of integration**.
- From geometrical point of view an indefinite integral is **collection of family of curves** each of which is obtained by translating one of the curves parallel to itself upwards or downwards along with y- axis.

### ASSIGNMENT

1. $\int (2^x + e^x)^2 dx$	2. $\int 3^{2x+5} dx$	3. $\int \frac{\sin x}{\sin 3x} dx$	4. $\int \frac{dx}{\cos(x-a)\cos(x-b)}$
5. $\int \sin^4 x dx$	6. $\int \tan^4 x dx$	7. $\int \sqrt{\tan x} dx$	8. $\int \sqrt{\cot x} dx$
9. $\int \sin^3 x \cos^4 x dx$	10. $\int \cos x \cos 2x \cos 3x dx$	11. $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$	12. $\int (a^x + b^x)^2 / (a^x b^x) dx$
13. $\int \frac{(\cos 2x - \cos 2\beta)}{(\cos x - \cos \beta)} dx$	14. $\int \sin^{-1}(\cos x) dx$	15. $\int \frac{dx}{(e^x + e^{-x})}$	16. $\int \sqrt{e^x - 1} dx$
17. $\int \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) dx$	18. $\int \cos^{-1}(\sec x - \tan x) dx$	19. $\int e^{\frac{x}{2}} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx$	20. $\int e^{\sin x} \sin 2\alpha dx$
21. $\int \frac{x^5}{(2-x^3)^4} dx$	22. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$	23. $\int e^{\frac{-x}{2}} \left(\sqrt{\frac{1-\sin x}{1+\cos x}}\right) dx$	24. $\int e^{-x} \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$
25. $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$	26. $\int \cos \sqrt{x} dx$	27. $\int (e^{a \log x} + e^{x \log a}) dx$	28. $\int \frac{\log(\log x)}{x} dx$
29. $\int (2x+1) \sqrt{x^2 + 4} dx$	30. $\int (2x-5) \sqrt{1-3x^2} dx$	31. $\int \log x \sin^{-1} x dx$	32. $\int \sin x \log(\sin x) dx$
33. $\int \frac{x^2+2x+3}{\sqrt{x^2+x-1}} dx$	34. $\int \frac{x^2+1}{\sqrt{x^2-5}} dx$	35. $\int . \sin^{-1} \sqrt{\frac{x}{a+x}} dx$	36. $\int \frac{x \tan^{-1} x}{(1+x^2)^{\frac{3}{2}}} dx$
37. $\int \frac{1}{\cos x (5-4 \sin x)} dx$	38. $\int \frac{1}{\sin x (3+2 \cos x)} dx$	39. $\int \cos^{-1}\left(\frac{1}{x}\right) dx$	40. $\int \frac{\sin^{-1} x}{\sqrt{1-x}} dx$
41. $\int \frac{1}{(\sin x + \sin 2x)} dx$	42. $\int \frac{1}{(x^{-1}+1)x} dx$	43. $\int (\sin^{-1} x)^2 dx$	44. $\int x^2 \tan^{-1} x dx$
45. $\int \frac{1}{x^8-x} dx$	46. $\int \frac{x^2}{16-x^6} dx$	47. $\int \frac{x \tan^{-1} x}{(1+x^2)^2} dx$	48. $\int e^{3x} \left(\frac{3+\tan x}{\cos x}\right) dx$

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|--------------------------------------------------------------|------------------------------------------------------------|--------------------------------------------------------------------|---------------------------------------------------------------------|
| 49. $\int \frac{x^5}{x^3 - 2x^2 - 5x + 6} dx$                | 50. $\int \frac{x^3 - 1}{(x+1)(x+2)(x+3)} dx$              | 51. $\int e^x (\frac{\sin 4x - 4}{1 - \cos 4x}) dx$                | 52. $\int \frac{\log x}{x^2} dx$                                    |
| 53. $\int \frac{1}{x^7 + x} dx$                              | 54. $\int \frac{1}{2e^{2x} + e^x - 1} dx$                  | 55. $\int (\tan \alpha + \tan^3 \alpha)(1 + \tan 3\alpha) d\alpha$ | 56. $\int \frac{3\tan\beta - \tan^3\beta}{1 - 3\tan^2\beta} d\beta$ |
| 57. $\int \frac{1}{\sin x + \cos x} dx$                      | 58. $\int \frac{\sin x}{\sin x + \cos x} dx$               | 59. $\int e^x (\frac{1+x \log x}{x}) dx$                           | 60. $\int (x^2 + x)e^x dx$                                          |
| 61. $\int \sqrt{\frac{a+x}{a-x}} dx$                         | 62. $\int \frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{a}\sqrt{x}} dx$ | 63. $\int \sin^4 x \cos^4 x dx$                                    | 64. $\int \frac{1}{3+2 \sin x + \cos x} dx$                         |
| 65. $\int x \sqrt{x^2 + a^2} dx$                             | 66. $\int x \sqrt{\frac{1-x}{1+x}} dx$                     | 67. $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$             | 68. $\int \frac{1}{\sin^3 x + \cos^3 x} dx$                         |
| 69. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$           | 70. $\int x \sqrt{1+x^4} dx$                               | 71. $\int \frac{1}{(a+b \cos x)} dx$                               | 72. $\int \frac{\cos x}{\cos 3x} dx$                                |
| 73. $\int \frac{\sin x}{\sin 4x} dx$                         | 74. $\int \frac{1}{(x^2+1)\sqrt{x}} dx$                    | 75. $\int \frac{1+\tan^2 x}{1-\tan^3 x} dx$                        | 76. $\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$                  |
| 77. $\int \tan^3 x dx$                                       | 78. $\int \sin^6 x dx$                                     | 79. $\int \frac{1}{\sin x + \tan x} dx$                            | 80. $\int x \sin 3x \cos 5x dx$                                     |
| 81. $\int \sec^5 x dx$                                       | 82. $\int \sec^2 x \operatorname{cosec}^2 x dx$            | 83. $\int \sqrt{x^2 - 10x + 29} dx$                                | 84. $\int \sqrt{15 + 4x - x^2} dx$                                  |
| 85. $\int \frac{\cos 2x}{\cos x \sin x} dx$                  | 86. $\int \frac{3\cos^3 x - 2\sin^3 x}{\sin x \cos x} dx$  | 87. $\int \frac{1}{\sqrt{x^2 - a^2}} dx$                           | 88. $\int \frac{1}{\sqrt{x^2 + a^2}} dx$                            |
| 89. $\int \frac{1}{(a \sin x + b \cos x)^2} dx$              | 90. $\int \frac{\cos x + \sin x}{1 + \sin 2x} dx$          | 91. $\int \frac{1}{x^2 - a^2} dx$                                  | 92. $\int \frac{1}{a^2 - x^2} dx$                                   |
| 93. $\int (\tan^n x + \tan^{n+2} x) dx$                      | 94. $\int \sqrt{\frac{\tan x}{\sin x \cos x}} dx$          | 95. $\int \sqrt{a^2 + x^2} dx$                                     | 96. $\int \sqrt{a^2 - x^2} dx$                                      |
| 97. $\int \frac{dx}{(\sin x + 2 \cos x)(3 \sin x - \cos x)}$ | 98. $\int \frac{\cos x}{\cos(x+\alpha)} dx$                | 99. $\int \sqrt{x^2 - a^2} dx$                                     | 100. $\int (x-2)\sqrt{1-2x-x^2} dx$                                 |

### FROM BOARD PAPERS

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|----------------------------------------------------------|---------------------------------------------|----------------------------------------|----------------------------------------------------|
| 1. $\int \frac{dx}{1+\sin x}$                            | 2. $\int \tan^{-1}(\frac{1-x}{1+x}) dx$     | 3. $\int \frac{xe^x}{x^2+1} dx$        | 4. $\int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x} dx$ |
| 5. $\int \sin^{-1}(3x-4x^3) dx$                          | 6. $\int e^x \operatorname{cosec}^2 e^x dx$ | 7. $\int \frac{2x}{(x^2+1)(x^2+2)} dx$ | 8. $\int \frac{2x+5}{(x^2-x-2)} dx$                |
| 9. $\int e^x (\sec x + \operatorname{sech} x \tan x) dx$ | 10. $\int \sqrt{1-x^2} dx$                  | 11. $\int \sec^4 x \tan x dx$          | 12. $\int \frac{\log x}{x^2} dx$                   |
| 13. $\int \frac{\sin 4x}{\sin x} dx$                     | 14. $\int e^x \cos x dx$                    | 15. $\int \frac{x}{x^4+x^2+1} dx$      | 16. $\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$ |

17.  $\int (x+1)(x \log x)^2 dx$     18.  $\int \cos 2x \log(\sin x) dx$     19.  $\int \frac{x^2 \tan^{-1} x}{1+x^2} dx$     20.  $\int \frac{\cos x}{(1+\sin 2x)(2+\sin x)} dx$
21.  $\int e^x \sec x (1+\tan x) dx$     22.  $\int \frac{x^2-1}{x^4+1} dx$     23.  $\int \frac{x \log x}{(x+2)(x-1)} dx$     24.  $\int (x^2-4x-3) dx$
25.  $\int \frac{(\sin x \cos x - 1)}{\sin^2 x} e^x dx$     26.  $\int x^2 \sin^{-1} x dx$     27.  $\int \frac{x^2+x-6}{(x-2)(x-1)} dx$     28.  $\int e^x (\tan x + \log \sin x) dx$
29.  $\int \frac{\sqrt{a-x}}{\sqrt{a+x}} dx$     30.  $\int \frac{dx}{\sqrt{x+a}\sqrt{x+b}} dx$     31.  $\int \frac{dx}{x \cos^2(1+\log x)} dx$     32.  $\int \frac{x-\sin x}{1-\cos x} dx$
33.  $\int \frac{3x+5}{x^3-x^2-x+1} dx$     34.  $\int \frac{e^x(2-\sin 2x)}{(1-\cos 2x)} dx$     35.  $\int \frac{x^2-1}{x^4+x^2+1} dx$     36.  $\int \frac{e^{2x}+1}{e^{2x}-1} dx$
37.  $\int x\sqrt{1+x-x^2} dx$     38.  $\int \frac{dx}{x+\sqrt{x}}$     39.  $\int \sin^{-1}(\frac{2x}{x^2+1}) dx$     40.  $\int \frac{x dx}{\sqrt{8+x-x^2}} dx$
41.  $\int \frac{2\sin 2\theta - \cos \theta}{6-\cos^2 \theta - 4\sin \theta} dx$     42.  $\int e^{-x} \sin^2 x dx$     43.  $\int \frac{dx}{(x-1)(x+1)(x+2)}$     44.  $\int (2x-1)(x-3)^{\frac{5}{3}} dx$
45.  $\int \frac{dx}{\sqrt{x-\alpha}\sqrt{x-\beta}}$     46.  $\int \frac{(2+\sin 2x)}{(1+\cos 2x)} e^x dx$     47.  $\int \sin^{-1} x dx$     48.  $\int \frac{dx}{1+x^4}$
49.  $\int \frac{dx}{\sqrt{x(1-2x)}}$     50.  $\int \sec x \log(\sec x + \tan x) dx$     51.  $\int \sec^{-1} x dx$     52.  $\int \frac{dx}{(1+\cos x)}$
53.  $\int \frac{x^4}{x^4-1} dx$     54.  $\int x \tan^{-1} x dx$     55.  $\int \frac{1-\tan x}{1+\tan x} dx$     56.  $\int \frac{x^2+2}{(x^2+1)(x^2+4)} dx$
57.  $\int \frac{x^2+1}{x^4-x^2+1} dx$     58.  $\int \frac{x^2}{(x-1)(x+1)^2} dx$     59.  $\int x^3 \log 2x dx$     60.  $\int \frac{e^x}{e^{2x}-4} dx$
61.  $\int x^3 e^x dx$     62.  $\int \frac{\sec^2(2\tan^{-1} x)}{1+x^2} dx$     63.  $\int \frac{dx}{9x^2+12x+13}$     64.  $\int \frac{dx}{x^3+x^2+x+1}$
65.  $\int \frac{dx}{\cos^2 x + 2\sin^2 x}$     66.  $\int \frac{dx}{\sqrt{1+\cos 2x}}$     67.  $\int \frac{x^2+1}{x^4+1} dx$     68.  $\int e^{ax} \cos(bx+c) dx$
69.  $\int \cos 4x \cos 3x dx$     70.  $\int \frac{dx}{\sqrt{7-6x-x^2}}$     71.  $\int x \tan^{-1} x dx$     72.  $\int \frac{5}{(x^2+1)(x+2)} dx$
73.  $\int (2x-5)\sqrt{x^2-4x+3} dx$     74.  $\int x^3 \cos x^4 dx$     75.  $\int \frac{e^x(1+\sin x \cos x)}{\cos^2 x} dx$     76.  $\int (2x+3)\sqrt{x^2+4x+3} dx$
77.  $\int (\cot x + \log(\sin x)) e^x dx$     78.  $\int \frac{dx}{(x^2+1)x}$     79.  $\int \frac{x}{\cos^2 x} dx$     80.  $\int \frac{dx}{\sqrt{2x+1}+\sqrt{2x+2}}$
81.  $\int \frac{dx}{(x^3+1)x}$     82.  $\int \frac{dx}{(x^8+1)x}$     83.  $\int \frac{dx}{x^2+6x+12}$     84.  $\int \frac{dx}{\sqrt{x^2-4x+2}}$
85.  $\int x \log(x+1) dx$     86.  $\int \frac{dx}{(x^6+1)x}$     87.  $\int \frac{dx}{\sqrt{x^2-6x+10}}$     88.  $\int x \cos^{-1} x dx$

$$89. \int e^x(\tan x + \log(\sec x)) dx$$

$$91. \int \frac{e^x(1+x)}{(2+x)^2} dx$$

$$92. \int \tan^{-1}(\sqrt{x}) dx$$

$$93. \int \sqrt{2ax - x^2} dx$$

$$94. \int \frac{dx}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$$

$$95. \int \log[x + \sqrt{x^2 + a^2}] dx$$

$$96. \int \sqrt{\frac{\cos 2x}{\cos x}} dx$$

$$97. \int \frac{dx}{(4+5 \sin x)}$$