



## DEFINITE INTEGRALS

Q.1)	$I = \int_0^3 \frac{1}{x^2(x+1)} dx$
Sol.1)	$I = \int_0^3 \frac{1}{x^2(x+1)} dx$ Type: partial fraction let $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$ $1 = A(x)(x + 1) + B(x + 1) + C(x^2)$ $1 = A(x^2 + x) + B(x + 1) + C(x^2)$ Comp. coff. of $x^2$ , $x$ & constant $0 = A + C$ $0 = A + B$ $1 = B$ Solving these equation we get $B = 1, A = -1, C = 1$ $\therefore I = \int_1^3 \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x+1} dx$ $= \left[ -\log x - \frac{1}{x} + \log(x+1) \right]_1^3$ $= \left[ \left( -\log 3 - \frac{1}{3} + \log 4 \right) - \left( -\log 1 - 1 + \log 2 \right) \right]$ $= -\log 3 - \frac{1}{3} + \log 4 + 0 + 1 - \log 2$ $= -\log 3 + \frac{2}{3} + 2 \log 2 - \log 2$ $= \frac{2}{3} + \log 2 - \log 3$ $I = \frac{2}{3} + \log \left(\frac{2}{3}\right)$ Ans ....
Q.2)	$I = \int_0^{\frac{\pi}{4}} \sqrt{1 - \sin 2x} dx$
Sol.2)	$I = \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x - 2\sin x \cos x} dx$ $I = \int_0^{\frac{\pi}{4}} \sqrt{(\cos x - \sin x)^2} dx$ ... ... $\begin{cases} 0 < x < \frac{\pi}{4} \\ \cos > \sin x \end{cases}$ $I = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx$ $I = (\sin x + \cos x)^{\frac{\pi}{4}}$



	$I = \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (0 + 1)$ $I = \sqrt{2} - 1 \quad \text{ans.}$
Q.3)	$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1 - \cos(2x)} dx$
Sol.3)	$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{1 - \cos(2x)} dx$ $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{\sin^2 x + \cos^2 x - 2 \sin x \cos x} dx$ $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{(\sin x - \cos x)^2} dx \quad \dots \dots \begin{cases} \frac{\pi}{4} < x < \frac{\pi}{2} \\ \sin x > \cos x \end{cases}$ $I = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x) dx$ $I = [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $I = -[\cos x + \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $I = - \left[ (0 + 1) - \left( \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right]$ $I = -1 + \sqrt{2}$ $I = \sqrt{2} - 1 \quad \text{Ans ....}$
Q.4)	$I = \int_0^{2\pi} e^x \cdot \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$
Sol.4)	$I = \int_0^{2\pi} e^x \cdot \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx \quad \{\text{type 1st Repeats of the two times by parts}\}$ $I = \left[ \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot e^x \right]_0^{2\pi} - \int_0^{2\pi} \frac{1}{2} \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot e^x dx$ $I = \left[ \sin\left(\pi + \frac{\pi}{4}\right) \cdot e^{2x} \sin\left(\frac{\pi}{4}\right) \cdot e^0 \right] - \frac{1}{2} \int_0^{2\pi} e^x \cdot \cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) dx$ $I = \left[ -\frac{1}{2} \cdot e^{2x} - \frac{1}{\sqrt{2}} \right] - \frac{1}{2} \left[ \left( \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\right) \cdot e^x \right)_0^{2\pi} + \frac{1}{2} \int_0^{2\pi} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) \cdot e^x dx \right]$ $I = \left( \frac{-e^{2x}-1}{\sqrt{2}} \right) - \frac{1}{2} \left[ \cos\left(\pi + \frac{\pi}{4}\right) \cdot e^{2x} - \cos\left(\frac{\pi}{4}\right) e^0 \right] - \frac{1}{4} \int_0^{2\pi} e^x \cdot \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx$ $I = \frac{-e^{2x}-1}{\sqrt{2}} - \frac{1}{2} \left[ -\frac{1}{\sqrt{2}} \cdot e^{2\pi} - \frac{1}{\sqrt{2}} \right] - \frac{1}{4} I$ $I + \frac{1}{4} I = \frac{-e^{2\pi}-1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} (e^{2\pi} + 1)$ $\frac{5I}{4} = \frac{-2e^{2\pi}-2+e^{2\pi}+1}{2\sqrt{2}}$ $\frac{5I}{4} = \frac{-(e^{2\pi}+1)}{2\sqrt{2}}$ $I = -\frac{4}{10\sqrt{2}} (e^{2\pi} + 1)$

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	$I = -\frac{2}{5\sqrt{2}}(e^{2\pi} + 1)$ $I = -\frac{\sqrt{2}}{5}(e^{2\pi} + 1) \quad \text{ans.}$
Q.5)	$I = \int_e^{e^2} \frac{1}{\log x} - \frac{1}{(\log x)^2} dx$
Sol.5)	$I = \int_e^{e^2} \frac{1}{\log x} - \frac{1}{(\log x)^2} dx$ <p>Put <math>\log x = t</math>      when <math>x = e ; t = 1</math>  <math>x = e^t</math>      when <math>x = e^2 ; t = 2</math>  <math>dx = e^t dt</math></p> $I = \int_1^2 e^t \left( \frac{1}{t} - \frac{1}{t^2} \right) dt$ $= \int_1^2 e^t \cdot \frac{1}{t} dt - \int_1^2 e^t \cdot \frac{1}{t^2} dt$ $= \left[ e^t \cdot \frac{1}{t} \right]_1^2 + \int_1^2 \frac{1}{t^2} e^t dt - \int_1^2 \frac{1}{t^2} e^t dt$ $= \left[ e^2 \cdot \frac{1}{2} \right] - [e]$ $I = \frac{e^2}{2} - e \quad \text{ans.}$
Q.6)	$\int_0^\pi \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$
Sol.6)	$\int_0^\pi \left( \sin^2 \frac{x}{2} - \cos^2 \frac{x}{2} \right) dx$ $= \int_0^\pi - \left( \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) dx$ $= \int_0^\pi -\cos x dx \quad \dots\dots [\cos 2\theta = \cos^2 \theta - \sin^2 \theta]$ $= -(sin)_0^\pi$ $= -(sin \pi - sin 0)$ $= 0 \quad \text{Ans.....}$ <p>(c) <math>I = \int_0^\infty \frac{x}{(1+x)(1+x^2)}</math></p> $\frac{\pi}{4} \quad \text{Hint: partial fraction (Type: 2)}$