

DEFINITE INTEGRALS

Q.1)	$I = \int_0^\pi \log\left(1 + \cos x\right) dx$
Sol.1)	$I = \int_0^{\pi} \log(1 + \cos x) dx$ (1)
	$I = \int_0^{\pi} \log \left[1 + \cos \left(\pi - x \right) \right] dx$ (P-IV)
	$I = \int_0^{\pi} \log (1 - \cos x) dx \qquad(2)$
	(1) + (2)
	$2I = \int_0^{\pi} \log\left((1+\cos x)(1-\cos x)\right) dx$
	$2I = \int_0^\pi \log\left(1 - \cos^2 x\right) dx$
	$2I = \int_0^\pi \log(\sin^2 x) dx$
	$2I = 2\int_0^{\pi} \log(\sin x) dx \qquad \qquad \dots [\log m^n = n \log m]$
	$I = \int_0^\pi \log(\sin x) dx$
	$I = 2 \int_0^{\frac{\pi}{2}} \log \left(\sin x \right) dx \qquad \dots \dots (P-VI)$
	$\frac{1}{2} = \int_0^{\frac{\pi}{2}} \log(\sin x) dx \qquad \dots (3)$ $\frac{1}{2} = \int_0^{\frac{\pi}{2}} \log\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx \qquad \dots (P-IV)$
	$\frac{1}{2} = \int_0^{\frac{\pi}{2}} \log \left(\sin \left(\frac{\pi}{2} - x \right) \right) dx \qquad \dots (P-IV)$
	$\frac{1}{2} = \int_0^{\frac{\pi}{2}} \log(\cos x) dx \qquad(4)$
	(3) + (4)
	$I = \int_0^{\frac{\pi}{2}} \log \left(\sin x \cdot \cos x \right) dx$
	$I = \int_0^{\frac{\pi}{2}} log \left(\frac{sin(2x)}{2} \right) dx$
	$I = \int_0^{\frac{\pi}{2}} \log \left(\sin(2x) \right) - \log 2 dx$
	$I = \int_0^{\frac{\pi}{2}} \log(\sin(2x)) dx - \int_0^{\frac{\pi}{2}} \log 2 dx$
	$I = \int_0^{\frac{\pi}{2}} \log(\sin(2x)) dx - \log_2(x)_0^{\frac{\pi}{2}}$
	$I = \int_0^{\frac{\pi}{2}} \log(\sin(2x)) dx - \frac{\pi}{2} \log 2$
	$I = I_1 - \frac{\pi}{2} \log 2 \qquad(5)$
	Where $I_1 = \int_0^{\frac{\pi}{2}} \log \left(\sin(2x) \right) dx$
	$Put \ 2x = t \qquad when \ x = 0 \ ; \ t = 0$
	$dx = \frac{dt}{2} \qquad \qquad x = \frac{\pi}{2} \; ; \; t = \pi$
	$\therefore I_1 = \frac{1}{2} \int_0^{\pi} \log(\sin t) dx$

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	$I_1 = \frac{1}{2} \times 2 \int_0^{\frac{\pi}{2}} \log(\sin t) dt$ (P-VI)		
	$I_1 = \int_0^{\frac{\pi}{2}} \log(\sin t) dt$		
	$I_1 = \int_0^{\frac{\pi}{2}} \log(\sin x) dx \qquad \qquad \dots \dots (P-I)$		
	$I_1 = \frac{I}{2} \qquad \dots \{from \ eq. 3\}$		
	$\therefore eq.(5)$ becomes		
	$I = \frac{I}{2} - \frac{\pi}{2} \log 2$		
	$\Rightarrow I = \frac{I}{2} = \frac{-\pi}{2} \log 2$		
	$\Rightarrow \frac{I}{2} = \frac{-\pi}{2} \log 2$		
	$I = -\pi \log 2$ ans.		
Q.2)	$I = \int_0^\pi \frac{x \tan x}{\sec x \cdot \csc x} dx$		
Sol.2)	$I = \int_0^\pi \frac{\frac{x \sin x}{\cos x}}{\frac{1}{\cos x} \cdot \frac{1}{\sin x}} dx$		
	$I = \int_0^\pi x \sin^2 dx \qquad \dots (1)$		
	$I = \int_0^{\pi} (\pi - x) \sin^2(\pi - x) dx$ (P-IV)		
	$I = \int_0^{\pi} (\pi - x) \sin^2 x dx \qquad(2)$		
	(1) + (2)		
	$2I = \int_0^{\pi} x \sin^2 x + \pi \sin^2 x - x \sin^2 x dx$		
	$2I = \pi \int_0^{\pi} \sin^2 x dx$		
	$2I = 2\pi \int_0^{\frac{\pi}{2}} \sin^2 dx \qquad(P-VI)$		
	$I = \pi \int_0^{\frac{\pi}{2}} \sin^2 x dx$		
	$I = \pi \int_0^{\frac{\pi}{2}} \frac{1 - \cos(2x)}{2} dx$		
	$I = \frac{\pi}{2} \left[x - \frac{\sin(2x)}{2} \right]_0^{\frac{\pi}{2}}$		
	$I = \frac{\pi}{2} \left[\left(\frac{\pi}{2} - \frac{\sin \pi}{2} \right) - (0 - 0) \right]$		
	$I = \frac{\pi}{2} \left[\frac{\pi}{2} - 0 \right]$		
	$I = \frac{\pi^2}{4} ans.$		
Q.3)	$\int_{1}^{2} \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$		
Sol.3)	$I = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx \qquad \dots $		
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	$I = \int_{1}^{2} \frac{\sqrt{1+2-x}}{\sqrt{3-(1+2-x)} + \sqrt{(1+2-x)}} dx \qquad \dots \dots \left[\int_{a}^{b} f(x) dx = \int_{a}^{b} f(a+b-x) dx \right]$	
	$I = \int_{1}^{2} \frac{\sqrt{3-x}}{\sqrt{x} + \sqrt{3-x}} dx \qquad(2)$	
	(1) + (2)	
	$2I = \int_1^2 \frac{\sqrt{x} + \sqrt{3-x}}{\sqrt{3-x} + \sqrt{x}} dx$	
	$2I = \int_1^2 1 \cdot dx$	
	$2I = (x)_1^2$	
	2I = 2 - 1	
	$\Rightarrow I = \frac{1}{2}$ ans.	
Q.4)	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx$	
Sol.4)	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx \qquad(1)$	
	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}} dx \qquad \dots (P-V) \text{ (above)}$	
	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\tan x}} dx$	
	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \frac{1}{\sqrt{\cot x}}} dx$	
	$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + 1} dx \qquad \dots (2)$	
	(1) + (2)	
	$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x} + 1}{\sqrt{\cot x + 1}} dx$	
	$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 \cdot dx$	
	$2I = (x)^{\frac{\pi}{3}}_{\frac{\pi}{6}}$	
	$2I = \frac{\pi}{3} - \frac{\pi}{6}$	
	$2I = \frac{\pi}{6}$	
	$I = \frac{\pi}{12}$ ans.	
Q.5)	$I = \int_0^1 \frac{ 5x - 3 }{(3/5)} dx$	
Sol.5)	$I = \int_0^1 \frac{ 5x - 3 }{(3/5)} dx$	
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	$I = -\int_0^{\frac{3}{5}} (5x - 3) dx + \int_{\frac{3}{5}}^{1} (5x - 3) dx$
	$I = -\left[\frac{5x^2}{2} - 3x\right]_0^{\frac{3}{5}} + \left[\frac{5x^2}{2} - 3x\right]_{\frac{3}{5}}^{1}$
	$I = -\left[\frac{5}{2} \cdot \frac{9}{25} - 3 \cdot \frac{3}{5}\right] + \left[\left(\frac{5}{2} - 3\right) - \left(\frac{5}{2} \cdot \frac{9}{25} - 3 \cdot \frac{3}{5}\right)\right]$
	$I = -\left(\frac{9}{10} - \frac{9}{5}\right) + \left[-\frac{1}{2} - \frac{9}{10} + \frac{9}{5}\right]$
	$I = -\frac{9}{10} + \frac{9}{5} - \frac{1}{2} - \frac{9}{10} + \frac{9}{5}$
	$I = \frac{13}{10} \qquad ans.$
Q.6)	$I = \int_{-5}^{5} \frac{ x-2 }{(2)} dx$
Sol.6)	$I = \int_{-5}^{5} \frac{ x-2 }{(2)} dx$
	$I = -\int_{5}^{2} (x-2) dx + \int_{2}^{5} (x-2) dx$
	$I = -\left[\frac{x^2}{2} - 2x\right]_{-5}^2 + \left[\frac{x^2}{2} - 2x\right]_2^5$
	$I = -\left[(2-4) - \left(\frac{25}{2} + 10\right) \right] + \left[\left(\frac{25}{2} - 10\right) - (2-4) \right]$
	$I = -\left[-2 - \frac{45}{2}\right] + \left[\frac{5}{2} + 2\right]$
	$I = 2 + \frac{45}{2} + \frac{5}{2} + 2 = 29$ ans.
Q.7)	$I = \int_{1}^{5} \frac{ x - 6 }{(6)} dx$
Sol.7)	$I = \int_{1}^{5} \frac{ x - 6 }{(6)} dx$
	$I = -\int_1^5 (x - 6) dx$
	$I = -\left[\frac{x^2}{2} - 6x\right]_1^5$
	$I = -\left[\left(\frac{25}{2} - 30 \right) - \left(\frac{1}{2} - 6 \right) \right]$
	$I = -\left[\frac{-35}{2} + \frac{11}{2}\right] = 12$ ans.
Q.8)	$I = \int_{-1}^{1} \frac{e x }{(0)} dx$
Sol.8)	$I = \int_{-1}^{0} e^{-x} dx + \int_{0}^{1} e^{x} dx$
	$I = \left[\frac{e^{-x}}{-1}\right]_{-1}^{0} + \left[e^{x}\right]_{0}^{1}$

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	$I = \left[\frac{1}{-1} - \frac{e^1}{-1}\right] + \left[e^1 - e^0\right]$
	I = [-1 + e] + [e - 1]
	I = 2e - 2 ans.
Q.9)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x + \cos x dx$
Sol.9)	$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x + \cos x dx$
	$I = \int_{-\frac{\pi}{2}}^{0} \sin(-x) + \cos(-x) dx + \int_{0}^{\frac{\pi}{2}} \sin(x) + \cos(x) dx$
	$I = \int_{-\pi/2}^{0} -\sin x + \cos x dx + \int_{0}^{\frac{\pi}{2}} \sin x + \cos x dx$
	$I = [\cos x + \sin x]_{-\frac{\pi}{2}}^{0} + [-\cos x + \sin x]_{0}^{\frac{\pi}{2}}$
	$I = \left[\left(\cos 0 + \sin 0 \right) - \left(\cos \left(-\frac{\pi}{2} \right) \right) + \sin \left(-\frac{\pi}{2} \right) \right] + \left[\left(-\cos \frac{\pi}{2} + \sin \frac{\pi}{2} \right) - \left(-\cos 0 + \sin 0 \right) \right]$
	I = [(1+0) - (0-1)] + [(0+1) - (-1+0)]
	$I = 2 + 2 = 4 \qquad \text{ans.}$
Q.10)	$I = \int_0^2 x^2 + 2x - 3 dx$
Sol.10)	$I = \int_0^2 \frac{ (x+3)(x-1) }{(-3)} dx$
	0 1 2
	$\therefore I = -\int_0^1 (x^2 + 2x - 3) dx + \int_1^2 (x^2 + 2x - 3)$
	$I = -\left[\frac{x^2}{3} + x^2 - 3x\right]_0^1 + \left[\frac{x^3}{3} + x^2 - 3x\right]_1^2$
	$I = -\left[\left(\frac{1}{3} + 1 - 3\right) - (0)\right] + \left[\left(\frac{8}{3} + 4 - 6\right) - \left(\frac{1}{3} + 1 - 3\right)\right]$
	$I = -\left[\frac{-5}{3}\right] + \left[\frac{2}{3} + \frac{5}{3}\right]$
	$I = \frac{12}{3} = 4$ ans.

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