TOPIC 8
DIFFERENTIAL EQUATIONS
SCHEMATIC DIAGRAM

		(ii).General and particular solutions of a differential equation	**	Ex. 2,3 pg384
		(iii).Formation of differential equation whose general solution is given	*	Q. 7,8,10 pg 391
		(iv).Solution of differential equation by the method of separation of variables	*	Q.4,6,10 pg 396
		(vi).Homogeneous differential equation of first order and first degree	**	Q. 3,6,12 pg 406
		(vii)Solution of differential equation of the type dy/dx +py=q where p and q are functions of x And solution of differential equation of the type dx/dy+px=q where p and q are functions of y	***	Q.4,5,10,14 pg 413,414

SOME IMPORTANT RESULTS/CONCEPTS

** Order of Differential Equation : Order of the heighest order derivative of the given differential equation is called the order of the differential equation.

** Degree of the Differential Equation : Heighest power of the heighest order derivative when powers of all the derivatives are of the given differential equation is called the degree of the differential equatin

** Homogeneous Differential Equation : $\frac{dy}{dx}=\frac{f_1(x,y)}{f_2(x,y)}$, where $f_1(x,y)$ & $f_2(x,y)$ be the homogeneous function of same degree.

** Linear Differential Equation :

i. $\frac{dy}{dx}+py=q$, where p & q be the function of x or constant.

Solution of the equation is : $y.e^{\int p\,dx}=\int e^{\int p\,dx}.q\,dx$, where $e^{\int p\,dx}$ is Integrating Factor (I.F.)

ii. $\frac{dx}{dy}+px=q$, where p & q be the function of y or constant.

Solution of the equation is: $x.e^{\int p\,dy}=\int e^{\int p\,dy}.q\,dy$, where $e^{\int p\,dy}$ is Integrating Factor (I.F.)

ASSIGNMENTS

1. *Order and degree of a differential equation*

LEVEL I

1. Write the order and degree of the following differential equations

(i) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$

2. *General and particular solutions of a differential equation*

LEVEL I

1. Show that $y = e^{-x} + ax + b$ is the solution of $e^x \frac{d^2y}{dx^2} = 1$

3. *Formation of differential equation*

LEVEL II

1. Obtain the differential equation by eliminating a and b from the equation y = e^x(acosx + bsinx)

LEVEL III

1. Find the differential equation of the family of circles $(x - a)^2 - (y - b)^2 = r^2$

2. Obtain the differential equation representing the family of parabola having vertex at the origin and axis along the positive direction of x-axis

4. *Solution of differential equation by the method of separation of variables*

LEVEL II

1. Solve $\frac{dy}{dx} = 1 + x + y + xy$

2. Solve $\frac{dy}{dx} = e^{-y}\cos x$ given that y(0)=0.

3. Solve $(1+x^2)\frac{dy}{dx} - x = \tan^{-1}x$

5. *Homogeneous differential equation of first order and first degree*

LEVEL II

1. Solve $(x^2 + xy)dy = (x^2 + y^2)dx$

LEVEL III

Show that the given differential equation is homogenous and solve it.

1. $(x - y)\frac{dy}{dx} = x + 2y$

2. $ydx + x\log(\frac{y}{x})dy - 2xdy = 0$

3.Solve $xdy - ydx = \sqrt{x^2 - y^2}dx$ 4.Solve $x^2 ydx - (x^3 + y^3)dy = 0$

5.Solve $xdy - ydx = \sqrt{(x^2 + y^2)}dx$ **CBSE2011** 6.Solve $\left(y + 3x^2\right)\frac{dx}{dy} = x$

7. Solve $x\ dy + (y - x^3)dx = 0$ **CBSE2011** 8.Solve $x\ dy + (y + 2x^2)dx = 0$

6. ***Linear Differential Equations***

LEVEL I

1.Find the integrating factor of the differential $x\frac{dy}{dx} - y = 2x^2$

LEVEL II

1.Solve $\frac{dy}{dx} + 2y\tan x = \sin x$ 2. Solve $(1+x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

3. Solve $x\frac{dy}{dx} + y = x\log x$

LEVEL III

1. Solve $\frac{dy}{dx} = \cos(x+y)$ 2.Solve $ye^y dx = (y^3 + 2xe^y)dy$

3. Solve $x^2\frac{dy}{dx} = y(x+y)$ 4. Solve $\frac{dy}{dx} + \frac{4x}{x^2+1}y = -\frac{1}{(x^2+1)^3}$

5. Solve the differential equation $\left(x + 2y^2\right)\frac{dy}{dx} = y$;given that when x=2,y=1

Questions for self evaluation

1. Write the order and degree of the differential equation $\left(\frac{d^3y}{dy^3}\right)^2 + \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$

2. Form the differential equation representing the family of ellipses having foci on x –axis and centre at origin .

3. Solve the differential equation : $(\tan^{-1} y - x)dy = (1 + y^2)dx$, given that y = 0 when x = 0.

4. Solve the differential equation :$xdy - y\,dx = \sqrt{x^2 + y^2}dx$

5. Solve the differential equation : $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$.

6. Solve the differential equation : $x^2 dy + (y^2 + xy)\,dx = 0$, $y(1) = 1$

7. Show that the differential equation $2y.e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous andfind its particular solution given that y(0) = 1 .

8. Find the particular solution of differential equation

$$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x,\ \text{given that}\ y\left(\frac{\pi}{2}\right) = 0\ .$$