DIFFERENTIAL EQUATION

KEY POINTS TO REMEMBER

- **Differential Equation**: Equation containing derivatives of a dependent variable with respect to an independent variable is called differential equation.
- **Order of a differential Equation** : The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.
- **Degree of a differential equation**: Highest power of highest order derivative present in the equation is called degree of differential equation where equation is a polynomial equation in differential co-efficient.
- **Formation of differential equation:** We differentiate the family of curve as many times as the number of arbitrary constants in the given family of curves. Now eliminate the arbitrary constants from these equations. After elimination the equation obtained is differential equation.
- **Solution of differential equation :**
  i Variable separable method
  ii Homogeneous differential equation
  iii Linear differential equation.

ASSIGNMENT

1. Find the differential equation corresponding to the function :
   i) $y = A \cos mx + B \sin mx$
   ii) $y = A e^{2x} + B e^{-2x}$
   iii) $x^2 + y^2 = 2ax$
   iv) $y = e^x (A \cos x + B \sin x)$
   v) $y = a \sin(bx + c)$
   vi) $y = A \cos (x + B)$
   vii) $y = ae^{2x} + be^{-x}$
   viii) $y = c(x - c)^2$
   ix) $(x + a)^2 - 2y^2 = a^2$

2. Form a differential equation of all non vertical lines in a plane $d^2y/dx^2 = 0$.
3. Form a differential equation of all the circles $(x - a)^2 + (y - b)^2 = c^2$ eliminating a and b.
4. Form a differential equation corresponding to all circles in the first quadrant which touch the coordinate axes.
5. Find the differential equation corresponding to all the parabolas having their axis of symmetry coincident with the axis of x.
6. Solve:
   i) $dy/dx + \dfrac{1 + \cos 2y}{1 - \cos 2y} = 0$
   ii) $dy/dx = \tan (x + y)$
   iii) $(x - 1)dy/dx = 2xy$ given that $y(2) = 1$
   iv) $x^2 dy + y(x + y)dx = 0$
   v) $x \log x \, dy/dx + y = \dfrac{2}{x} \log x$

vi) $dy/dx = \frac{x + 1}{2 - y}$

7. Solve:

i) $(x^2 - 1)\, dy/dx + 2(x + 2)y = 2(x + 1)$

ii) $(\sin^4 x + \cos^4 x) dy/dx = 1$

iii) $\sqrt{(1 + x^2 + y^2 + x^2y^2)} + xy\, dy/dx = 0$

iv) $dy/dx + \frac{4xy}{x^2 + 1} = \frac{-1}{(x^2 + 1)^3}$

v) $(x^3 + y^3)dy - x^2y\, dx = 0$

vi) $(x \sin (y/x))dy = (y \sin y/x) - x)$

vii) $\cos^3 x\, dy/dx + y \cos x = \sin x$

viii) $(2x - 10y^3)dy + y\, dx = 0$

ix) $(y \sin 2x)dx - (1 + y^2 + \cos^2 x)dy = 0$

x) $(x + 2y^3)dy = y^2\, dx$

xi) $y\, dx + (x - y^3)\, dy = 0$

xii) $y e^y\, dx = (y^3 + 2x e^y)dy$ given that $y(0) = 1$

xiii) $(1 + y^2) + (x - e^{\tan y}\, dy/dx) = 0$

xiv) $dy/dx + x \sin 2y = x^3 \cos^2 y$

xv) $xy \log \left(\frac{x}{y}\right) dx + \{y^2 - x^2 \log \left(\frac{x}{y}\right)\}\, dy = 0$

xvi) $dy/dx = (4x + y + 1)^2$

xvii) $dy/dx = \cos(x + y) + \sin (x + y)$

xviii) $(3xy + y^2)\, dx + (x^2 + xy)\, dy = 0$