

CONTINUITY AND DIFFERENTIABILITY

KEY POINTS TO REMEMBER

- A function $f(x)$ is said to be **continuous at $x = c$** where $c \in D_f$ iff $\lim_{x \rightarrow c} f(x) = f(c)$.
- $f(x)$ is **continuous in $[a, b]$** iff f is continuous in (a, b) , $\lim_{x \rightarrow a^+} f(x) = f(a)$ and $\lim_{x \rightarrow b^-} f(x) = f(b)$.
- A real function f is said to be **continuous** if it is continuous at every point in the domain of f .
- Every Polynomial function is continuous in their respective domains.
- Trigonometric functions are continuous in their respective domains.
- $f(x) = [x]$ is discontinuous at all integral points and continuous for all $x \in R - Z$.
- Suppose f & g are are real valued functions such that $(f \circ g)$ is defined at c . If g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c .
- A function f is said to be differentiable at a point c iff $\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} = \lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c}$ ie **LHD(c)=RHD(c)** and value of above limit is denoted by $f'(c)$ and is called **derivative** of $f(x)$ at $x = c$.
- $\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(\log x) = \frac{1}{x}$
- Every differentiable function is continuous but converse need not be true.
- If $y = f(t)$, $x = g(t)$ then $\frac{dy}{dt} = \frac{f'(t)}{g'(t)}$.
- If $y = f(u)$ and $u = g(t)$ then $\frac{dy}{dt} = \frac{dy}{du} \frac{du}{dt} = f'(u) \cdot g'(t)$. **(Chain Rule)**
- **Rolle's theorem:** If $f(x)$ is continuous in $[a, b]$, derivable in (a, b) and $f(a) = f(b)$, then there exist atleast one real number $c \in (a, b)$ such that $f'(c) = 0$.

Mean Value Theorem : If $f(x)$ is continuous in $[a, b]$, derivable in (a, b) , then there exist atleast one real number $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

DIFFERENTIATION ASSIGNMENT (I)

Q1. Differentiate the following :

$$\begin{array}{llll}
 (1) \sqrt{ac + b} \sqrt{ax + b} & (2) 3x^{\frac{5}{3}} + 5x^{\frac{2}{5}} + x^{\frac{1}{7}} & (3) x^4 + x^2 \sqrt{x} + x\sqrt{x} + \frac{1}{\sqrt{x}} & (4) \frac{3x^3 + 5x^2 + 9}{x^4} \\
 (5) \frac{x^3 - 2x}{x+7} & (6) \frac{\frac{1}{x^{\frac{2}{3}}} + a^{\frac{1}{3}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}} & (7) \frac{(x+1)^2}{(x-2)^3} & (8) \frac{\sqrt{x}-1}{\sqrt{x}+1} \\
 (9) \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} & (10) \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} & (11) \frac{x}{1+\sin^2 x} & (12) \frac{1-x \cos x}{1+x \cos x}
 \end{array}$$

(13) $\frac{\sqrt{1-\sin x}}{\sqrt{1+\sin x}}$	(14) $\frac{\sqrt{\sec x-1}}{\sqrt{\sec x+1}}$	(15) $\tan(2x-1)$	(16) $\tan^2 x$
(17) $\tan(x^2+1)$	(18) $\sqrt{\cos x}$	(19) $\sin \sqrt{x}$	(20) $\sin(2x^2 + 3x + 7)$
(21) $\cos \sqrt{1-x^2}$	(22) $\tan(1-x^2)$	(23) $\cot(\sin \sqrt{x})$	(24) $\cos(\cos x^2)$
(25) $\cos^n x^2$	(26) $\sin^n x^n$	(27) $\sqrt{\cos \sqrt{x}}$	(28) $\sqrt{\sin x^2}$
(29) $\sqrt{\tan \sqrt{x^2+1}}$	(30) $\sin^2 x \cos^3 x$	(31) $\tan^3 x^2 \sec^2 x^2$	(32) $\sin x \sin 2x \sin 3x$
(33) $x^2 \sin\left(\frac{1}{x}\right)$	(34) $x^2 \tan\left(\frac{x}{2}\right)$	(35) $(1-x^2)^3 \cot^3 x$	(36) $\sqrt{1-x^2} \sin^2 x$
(37) $\cos^{-1} x^2$	(38) $\tan^{-1} \sqrt{x}$	(39) $\sqrt{\tan^{-1} x}$	(40) $\sin^{-1}\left(\frac{x}{a}\right)$
(41) $\tan^{-1} x^2$	(42) $\sin^{-1} \sqrt{x}$	(43) $\sec^{-1} 2x$	(44) $\tan^{-1}(\sec x)$
(45) $\cot(\cos^{-1} x)$	(46) $\sin^{-1}(\sqrt{\cos x})$	(47) $\sin(2 \sin^{-1} x)$	(48) $\tan^{-1}(\sin x + \cos x)$
(49) $(\sin^{-1} x^4)^4$	(50) $(\tan^{-1} x^3)^2$	(51) $(\operatorname{cosec}^{-1} \sqrt{x})^2$	(52) $\sqrt{\sin^{-1} \sqrt{x}}$
(53) $\sqrt{\sec^{-1} x^2}$	(54) $\frac{1}{(\tan^{-1} x)^2}$	(55) $\sqrt{1-x^2} \sin^{-1} x - x$	(56) $\frac{x}{(\tan^{-1} x)}$
(57) $\frac{(\cos^{-1} x)^2}{x}$	(58) $\frac{x \sin^{-1} x}{\sqrt{1-x^2}}$	(59) $(\sqrt{x} + \frac{1}{\sqrt{x}})^2$	(60) $(x + \sqrt{1-x^2}) \cos^{-1} x$
(61) $\cos^{-1} \sqrt{\frac{1+\cos x}{2}}$	(62) $\sin^{-1} \sqrt{\frac{1-\cos x}{2}}$	(63) $\tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$	(64) $\cot^{-1} \sqrt{\frac{1+\cos 3x}{1-\cos 3x}}$
(65) $\tan^{-1} \left(\frac{1-\cos x}{\sin x} \right)$	(66) $\tan^{-1} \left(\frac{\cos x}{1+\sin x} \right)$	(67) $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$	(68) $\tan \frac{\sqrt{1-\sin x}}{\sqrt{1+\sin x}}$
(69) $\tan^{-1}(\sec x + \tan x)$	(70) $\tan^{-1}(\operatorname{cosec} x - \cot x)$	(71) $\cot^{-1} \left(\frac{1+\sin x}{\cos x} \right)$	(72) $\cot^{-1} \left(\frac{\sin x}{1+\cos x} \right)$
(73) $\tan^{-1} \left(\frac{2x}{1-x^2} \right)$	(74) $\sin^{-1}(3x - 4x^3)$	(75) $\cos^{-1}(3x - 4x^3)$	(76) $\cos^{-1}(2x \sqrt{1-x^2})$
(77) $\cos^{-1}(1-2x^2)$	(78) $\tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$	(79) $\tan^{-1} \left(\frac{1}{\sqrt{x^2-1}} \right)$	(80) $\sin^{-1} \left(\frac{x}{\sqrt{x^2+a^2}} \right)$
(81) $\tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$	(82) $\tan^{-1} \left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right)$	(83) $\tan^{-1} \left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} \right)$	(84) $\sin^{-1} \left(\frac{5x+12\sqrt{1-x^2}}{13} \right)$
(85) $\cot^{-1} \left(\frac{1-x}{1+x} \right)$	(86) $\cos^{-1} \frac{\sqrt{1+x}}{\sqrt{2}}$		

Q2. If $y = b \tan^{-1} \left(\frac{x}{a} + \tan^{-1} \frac{y}{x} \right)$, find $\frac{dy}{dx}$.

Q3. Differentiate : (1) $\frac{5x}{\sqrt[3]{1-x^2}} + \sin^2(2x+3)$ (2) $\log(\sin \sqrt{x^2+1})$ (3) $\log \left(\frac{\sqrt{1+\cos^2 x}}{\sqrt{1-e^{2x}}} \right)$

$$(4) \log \left(\frac{\sqrt{1+\sin^2 x}}{\sqrt{1-\tan x}} \right)$$

Q4. Find $\frac{dy}{dx}$:

$$(1) x^3 + 3ax^2y + 3bxy^2 + y^3 = 5 \quad (2) x^2y + xy^2 = a^2 \quad (3) \frac{x^2}{y} + xy = \frac{3}{2} \quad (4) \frac{x}{y} + \frac{y}{x} = \frac{a}{y}$$

$$(5) \frac{x^2}{y^2} + \frac{x}{y} = 1 \quad (6) \cos(x+y) = y \sin x \quad (7) \tan(x+y) + \tan(x-y) = 1 \quad (8) \sin(xy) + \frac{x}{y} = x^2 - y$$

$$(9) \sin^2 x + x \cos y + xy = 0 \quad (10) y \sec x + \tan x + x^2y = 0 \quad (11) x^2y + xy^2 = 6(x^2 + y^2)$$

Q5. If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots}}}$, then prove that $(2y-1)\frac{dy}{dx} + \sin x = 0$

Q6. If $y = \frac{\sqrt{1-\sin 2x}}{\sqrt{1+\sin 2x}}$, then prove that $\frac{dy}{dx} + \sec^2(\frac{\pi}{4} - x) = 0$

Q7. If $y = \frac{\sqrt{\sec x + \tan x}}{\sqrt{\sec x - \tan x}}$, then prove that $\frac{dy}{dx} = \sec x (\sec x + \tan x)$

Q8. If $y = x \sin y$, prove that $x \frac{dy}{dx} = \frac{y}{1-x \cos y}$.

Q9. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\frac{dy}{dx} = \frac{y^2 - 2xy - 2x}{x^2 - 2xy - 2y}$

Q10. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, prove that $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$

Q11. If $x^2 + y^2 = t - t^{-1}$ and $x^4 + y^4 = t^2 + t^{-2}$, prove that $x^3 y \frac{dy}{dx} = 1$

Q12. If $y = \sqrt{x} + \frac{1}{2\sqrt{x}}$, prove that $2x \frac{dy}{dx} + y = 2\sqrt{x}$

Q13. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \infty$, prove that $\frac{dy}{dx} = y$

DIFFERENTIATION ASSIGNMENT (II)

1. Find $\frac{dy}{dx}$ for the following:

i. $x = \frac{2t}{1+t^2}$, $y = \frac{1-t^2}{1+t^2}$ ii. $x = \frac{3at}{1+t^3}$, $y = \frac{3at^2}{1+t^3}$ iii. $x = a \cos^3 t$, $y = a \sin^3 t$

iv. $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ v. $x = 2 \cos t - \cos 2t$, $y = 2 \sin t - \sin 2t$

vi. $x = a(\cos t + \sin t)$, $y = a(\sin t - \cos t)$ vii. $x = \frac{1}{1+\sin t}$, $y = \frac{1}{1-\sin t}$

viii. $x = \frac{\sin t}{1+\cos t}$, $y = \frac{\cot t}{1+\sin t}$

2. If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

3. If $x = 2 \cos t - \cos 2t$, $y = 2 \sin t - \sin 2t$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$.

4. If $y = x + \tan x$, prove $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$.

5. If $y = \tan^{-1} x$, prove $(1+x^2) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0$.

6. If $y = \tan x + \sec x$, prove that $(1 - \sin x) \frac{d^2y}{dx^2} = \cos x$.

7. If $y = 3\cos(\log x) + 4 \sin(\log x)$, show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$.

8. Prove that $\frac{d}{dx} \left(\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right) = \sqrt{a^2 - x^2}$

9. Differentiate the following functions:

i. $\log \sqrt{\frac{1-\cos x}{1+\cos x}}$ ii. $\log (\sin \sqrt{x^2 + 1})$ iii. $\log \sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$

iv. $y = \sqrt{x^2 + 1} - \log \left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \right)$ v. $\sqrt{\frac{\tan^{-1} x (x^2+1)}{\sin x^3}}$

10. If $y \sqrt{x^2 + 1} = \log (\sqrt{x^2 + 1} - x)$, prove that $(x^2 + 1) \frac{dy}{dx} + xy + 1 = 0$.

11. If $y = \log \left(\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right)$, show that $\frac{dy}{dx} - \sec x = 0$.

12. Differentiate the following:

i. $e^{\sin x} + (\tan x)^x$

ii. $x^{\sin x}$

iii. $x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$

iv. $(\sin x)^{\cos^{-1} x}$

v. $x^x + (\sin x)^x$

13. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

14. If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = y/x$

15. Find the derivative of the following functions:

i. $x = a(t + \sin t)$, $y = a(1 - \cos t)$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{2}$.

ii. $x = a \sin 2t (1 + \cos 2t)$, $y = b \cos 2t (1 - \cos 2t)$, then show that y' at $t = \frac{\pi}{4}$ is b/a .

iii. $x = a(\cos t + \log \tan t/2)$, $y = a \sin t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

16. If $x = a \frac{(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^4}$ find $\frac{dy}{dx}$.

17. If $x = \frac{a(1+t^2)}{1-t^2}$, $y = \frac{2t}{1-t^2}$ find $\frac{dy}{dx}$.

18. Find $\frac{d^2y}{dx^2}$ if $y = \log\left(\frac{x^2}{e^x}\right)$

19. If $y = \cot x$, prove that $\frac{d^2y}{dx^2} + 2y \frac{dy}{dx} = 0$.

20. If $y = x + \tan x$, prove that $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$.

21. If $y = \tan x + \sec x$, prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$

22. If $y = \sec x - \tan x$, prove that $\cos x \frac{d^2y}{dx^2} = y^2$

23. If $y = ae^{2x} + be^{-x}$, prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

24. If $y = \log(x + \sqrt{x^2 + 1})^2$, show that $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 2 = 0$.

25. If $y = \sin(\log x)$, prove that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$.

26. If $y = x \log\left(\frac{x}{a+bx}\right)$, prove that $\frac{d^2y}{dx^2} = \frac{1}{x} \left(\frac{a}{a+bx}\right)^2$

27. If $x = at^2$, $y = 2at$, find $\frac{d^2y}{dx^2}$.

28. If $x = a(t + \sin t)$, $y = a(1 - \cos t)$, find $\frac{d^2y}{dx^2}$.

DIFFERENTIATION ASSIGNMENT (III)

1. Differentiate:-

- $$(1) \log_{10} \sin x \quad (2) \sin^{-1} \left(\frac{2 \sin x + 3 \cos x}{\sqrt{13}} \right) \quad (3) \tan^{-1} \left(\frac{a \sin x + b \cos x}{a \cos x - b \sin x} \right) \quad (4) \tan^{-1} \left(\frac{5x}{1-6x^2} \right)$$
- $$(5) \cos^{-1} \left(\frac{3+5 \cos x}{5+3 \cos x} \right) \quad (6) \tan^{-1} \left(\frac{x^{\frac{1}{3}} + a^{\frac{1}{3}}}{1-x^{\frac{1}{3}} a^{\frac{1}{3}}} \right) \quad (7) \sec^{-1} \left(\frac{\sqrt{x}+1}{\sqrt{x}-1} \right) + \sin^{-1} \left(\frac{\sqrt{x}-1}{\sqrt{x}+1} \right) \quad (8) \tan^{-1} \left(\frac{x+1}{x-1} \right) + \tan^{-1} \left(\frac{x-1}{x+1} \right)$$
- $$(9) \sin^{-1} \left(\frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right) \quad (10) \log \{ (\cos x)^x + e^{-x^2} \} \quad (11) x = e^\theta (2 \cos \theta + \cos 2\theta), y = e^\theta (2 \sin \theta + \sin 2\theta)$$
- $$(12) \sin x \quad (13) \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}} \quad (14) \tan^{-1} \left(\frac{ax+b}{a+bx} \right) \quad (15) \sqrt{a + \sqrt{a+x}}$$
- $$(16) x^y y^x \quad (17) e^x \log \sqrt{x} \tan x \quad (18) \sin(\sin x^2) \text{ at } \sqrt{\frac{\pi}{2}} \quad (19) \tan^{-1} \left(\frac{\sqrt{1+a^2 x^2} - 1}{ax} \right)$$
- $$(20) \log(\sqrt{\sin x - \cos x}) \quad (21) e^x + e^y = e^{x+y}, \frac{dy}{dx}=? \quad (22) \log(\sqrt{x^2 + y^2}) = \tan^{-1} \frac{y}{x}, \frac{dy}{dx}=?$$
- $$(23) \tan(x + 45^\circ)$$

2. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3 (x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$.

3. If $y = e^x \tan^{-1} x$, prove that $(1+x^2) \frac{d^2y}{dx^2} - 2(1-x-x^2) \frac{dy}{dx} + (1-x^2)y = 0$.

4. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that $\frac{d^2y}{dx^2} = \frac{-b^4}{a^2 y^3}$.

5. If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2) \frac{dy}{dx} + y = 0$.

6. If $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right)$, show that $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$.

7. If $y = e^{ax} \sin bx$, prove that $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$.

8. If $y = x^x$, show that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$.

9. If $y = \sin^{-1} (x^2 \sqrt{1-x^2} + x \sqrt{1-x^4})$, prove that $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$.

10. If $x = \sin \left(\frac{1}{a} \log y \right)$, show that $(1-x^2)y_2 - xy_1 - a^2 y = 0$.

11. If $y = f \left(\frac{2x-1}{x^2+1} \right)$, $f'(x) = \sin x^2$, find $\frac{dy}{dx}$.

12. If $y = \sqrt{a^2 - x^2}$, prove that $\frac{dy}{dx} + x = 0$.

13. If $xy = 4$, prove that $x \left(\frac{dy}{dx} + y^2 \right) = 3y$.
14. Find $\frac{dy}{dx}$: (1) $y = \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$ (2) $y = \tan^{-1}\left(\frac{2ax}{1-a^2x}\right)$
15. If $\cos^{-1}\frac{x^2-y^2}{x^2+y^2} = \tan^{-1}a$, prove that $\frac{dy}{dx} = \frac{y}{x}$.
16. Find $\frac{dy}{dx}$, if $\tan^{-1}(x^2 + y^2) = a$.
17. (1) If $xy = 1$, prove that $\frac{dy}{dx} + y^2 = 0$ (2) If $xy^2 = 1$, prove that $2\frac{dy}{dx} + y^3 = 0$.
18. If $x \sin(a+y) + \sin a \cos(a+y) = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.
19. If $y = (\log_{\cos x} \sin x)(\log_{\sin x} \cos x)^{-1} + \sin^{-1}\frac{2x}{1+x^2}$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$.
20. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.
21. If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} = -\frac{e^x}{e^y} \left(\frac{e^y-1}{e^x-1} \right)$.
22. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} + 1$, prove that $\frac{dy}{dx} = \frac{y}{x} \left\{ \frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x} \right\}$.
23. Given that $\cos\frac{x}{2} \cdot \cos\frac{x}{4} \cdot \cos\frac{x}{8} \cdot \dots = \frac{\sin x}{x}$, prove that $\frac{1}{2^2} \sec^2\frac{x}{2} + \frac{1}{2^4} \sec^2\frac{x}{4} + \dots = \operatorname{cosec}^2 x - \frac{1}{x^2}$.
24. If $x^3 y^7 = (x+y)^{20}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.
25. If $y = (\sin x - \cos x)^{\sin x - \cos x}$, find $\frac{dy}{dx}$.
26. If $x = \sin^{-1}\frac{2t}{1+t^2}$, $y = \tan^{-1}\frac{2t}{1-t^2}$, prove that $\frac{dy}{dx} = -1$.
27. If $x = 2 \cos\theta - \cos 2\theta$, $y = 2 \sin\theta - \sin 2\theta$, prove that $\frac{dy}{dx} = \tan\frac{3\theta}{2}$.
28. If $f(x) = \log_e(\log_e x)$, find $f'(e)$.
29. If $f(x) = x + 1$, find $\frac{d}{dx}(f \circ f)(x)$.
30. If $f'(1) = 2$, $y = f(\log_e x)$, find $\frac{dy}{dx}$ at $x = e$.
31. If $f(1) = 4$, $f'(1) = 2$, find $\log(f(e^x))$ with respect to x at $x = 0$.
32. If $y = \operatorname{cosec}^{-1} x$, show that $x(x^2 - 1) \frac{d^2y}{dx^2} + (2x^2 - 1) \frac{dy}{dx} = 0$.
33. If $y = \sin(\log x)$, prove that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$.
34. If $x = \sin\left(\frac{1}{a} \log y\right)$, show that $(1-x^2)y_2 - x y_1 - a^2 y = 0$.
35. If $y = \log(1 + \cos x)$, prove that $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} \cdot \frac{dy}{dx} = 0$.