Determinants

Important points to remember

- **Determinant :** To every square matrix A = $[A_{ij}]$ of order n x n, we can associate a number (real or complex) called determinant of A . It is denoted by det A or $|A|$.

  Properties:

  (1)$|AB|$ = $|A||B|$

  (2) $|kA|_{nxn}$ = k^n $|A|_{nxn}$,where k is a scalar.

  (3) Area of a triangle with vertices (x_1,y_1) , (x_2 , y_2) ,(x_3,y_3) is given by

  $$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

  (4) If the points (x_1,y_1) , (x_2 , y_2) ,(x_3,y_3) are collinear then

  $$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

  Adjoint of square matrix A is the transpose of the matrix whose elements have been replaced by their co-factors and is denoted as adj. A .

  Let A = $[a_{ij}]_{nxn}$

  adj A = $[A_{ij}]_{nxn}$

  Properties:

  (i) A (adjA) =(adjA)A = $|A|$I

  (ii)If A is a square matrix of order n then $|adj\ A| = |A|^{n-1}$.

  (iii)adj (AB) = (ad j B)(adj A).

  singular Matrix: A square matrix is called singular if $|A|$ = 0 , otherwise it will be called a non - singular matrix.

  Inverse of a matrix :A square matrix whose inverse exists, is called invertible matrix. inverse of a non-invertible matrix exists.

  Inverse of a matrix A is denoted by A^{-1} and is given by

  $$A^{-1} = \frac{1}{|A|} .adj\ A$$

Properties

(i) A A^{-1} = A^{-1}A = I

(ii)$(A^{-1})^{-1}$ =A

(iii)$(AB)^{-1} = B^{-1}A^{-1}$

(iv)$(A^T)^{-1} = (A^{-1})^T$

Solution of system of equations using matrix :

If AX =B is a matrix equation then its solution is x = A^{-1}B.

(i) If $|A| \neq 0$, system is consistent and has a unique solution.

(ii) If $|A| = 0$ and (adj A)B $\neq 0$, system is inconsistent and has no solution.

(iii) If $|A| = 0$ and (adj A)B$= 0$, system is consistent and has infinite solution.

ASSIGNMENT

1. If A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, Find A^{-1} and hence solve system of equation

x+2y-3z = - 4

2x +3y+2z = 2

3x-3y-4z =11

2. If A= $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, B= $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ Find AB & hence solve

x-y = 3,

2x+3y+4z =17

y +2z = 7.

3. Check the consistency and inconsistency of the following linear equations.

i) 3x - y +2z =3

2x +y +3z =5

x - 2y - z =1

ii) 2x +y-2z = 4

x- 2y +z =-2

5x-5y +z = -2

iii) $3x + y = 5$

$-6x - 2y = 9$

iv) $x + y - z = 0$

$x - 2y + z = 0$

$3x + 6y - 5z = 0$

4. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations.

$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$

5. Without expanding , show that the values of each of the following determinants is zero.

i) $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$ $\qquad$ ii) $\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

6. Using the properties of determinants evaluate the following:

i) $\begin{vmatrix} x+y & x & x \\ x & x+y & x \\ x & x & x+y \end{vmatrix}$ $\qquad$ ii) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

iii) $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ $\qquad$ iv) $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

7. Prove the following identities:

i) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

ii) $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$

iii) $\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$

iv) $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc-a^3-b^3-c^3$

v) $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$ = -(a-b)(b-c)(c-a)($a^2+b^2+c^2$)

vi) $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

vii) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

viii) $\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix}$

= a b c $(b^2+c^2+a^2)^3$

8. Determine the product $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$ and use it to solve the system of equations.

x + y + z = 0 , x + 2y-3z = -14, 2x -y+3z =9

9. Using the properties of determinants, prove the following:

$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ = xyz + x y +yz +zx (CBSE-2009)

10. If x, y, z all are different &

$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$ = 0, show that xyz = -1

11. If A & B are square matrices of order 3 such that $|A| = -1$ & $|B|$=3, then find the value of $|3AB|$.

12. Without expanding evaluate the determinant

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (a^y + a^{-y})^2 & (a^y - a^{-y})^2 & 1 \\ (a^z + a^{-z})^2 & (a^z - a^{-z})^2 & 1 \end{vmatrix}$$,where a > 0 and x,y,z ε R.

13. Using the properties of determinants, prove the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$$ [CBSE-2009]

14. Prove: $$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ac \end{vmatrix} = 0$$

15.Show that: $$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$$, where a ,b ,c, are in A.P.

16. Find the area of triangle using the determinants if three of its vertices are (5,2), (-3,-1),(6,0).

17. If the points (a ,b),(c , d),and (a+c, b+d) are collinear, show that

a d = b c .

18. Find the value of α so that the points (1,-5), (-4,5),and (α,7) are collinear.

19.If a, b, & c are distinct real no. and the system of equations

$a x + a^2y + (a^3+1)z = 0$

$b x + b^2y + (b^3+1)z = 0$

$c x + c^2y + (c^3+1)z = 0$ has a non trivial solution show that abc = -1.

20. Find the minor of element 5, $$\begin{vmatrix} -3 & 6 & 5 \\ 2 & 1 & 0 \\ -1 & 6 & 5 \end{vmatrix}$$

21. Find the co- factor of element a_{23}, $$\begin{vmatrix} -8 & 6 & 0 \\ 6 & 1 & 0 \\ -1 & 6 & 5 \end{vmatrix}$$

22.Using the co-factor of the second row of determinant

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$ find the value of Δ.

23.Find A^{-1} where $\begin{bmatrix} 6 & 4 & 2 \\ -12 & 15 & 18 \\ 25 & -20 & 15 \end{bmatrix}$ and solve the following

6x -12y +25z = 4

4x + 15y - 20z = 3

2x + 18y + 15z = 10

24. .Find A^{-1} where $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ and solve the following

3x +4y +7z =14

2x - y +3z = 4

x + 2y -3z = 0

25. solve the following system of equations using matrices:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ x , y, z≠0.

26. Find the value of α ,for which the homogeneous system of equation:

2x +3y -2z = 0

2x - y +3z = 0

7 x + α y - z = 0 has non trivial solutions. Find the solutions.

27.Solve the following determinant equation:

(i) $\begin{vmatrix} a+x & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

(ii) $\begin{vmatrix} 1+x & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$

ANSWERS:

4. x =3,y =-2,z=-1 **8.** x = -1 , y = -2 , z = 3 **11.** -81 23. x = 1/2, y = 1/3 , z = 1/5
24. x = 1, y = 1 , z = 1 **25.** x = 2, y = 3 , z = 5 **26.** $\alpha = \frac{57}{10}$, y = $\frac{5}{4}$ k , x = $\frac{-7}{8}$k, z = k