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## Determinants <br> Class 12 ${ }^{\text {th }}$

| Q.1) | Find the value of $x$ if the area of $\Delta$ is 35 square units with vertices ( $\mathrm{x}, 4),(2,-6)$ and $(5,4)$. |
| :---: | :---: |
| Sol.1) | $\begin{aligned} & \text { Let vertices are } A(x, 4), B(2,-6) \text { and } C(5,4) \\ & \text { Area of } \triangle \mathrm{ABC}=\frac{1}{2}\left\|\begin{array}{ccc} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{array}\right\| \\ & \quad 35=\frac{1}{2}\|x(-10)-4(-3)+1(38)\| \\ & \Rightarrow 35=\frac{1}{2}\|-10 x+12+38\| \\ & \Rightarrow 70=\|-10 x+50\| \\ & 70=-10 x+50 \left\lvert\, \begin{array}{l} -70=-10 x+50 \\ 10 x=-20 \\ \quad x=-2 \\ \therefore \quad x=-2, x=12 \quad \end{array}\right. \\ & \therefore \quad x=12 \\ & \text { ans. } \end{aligned}$ |
| Q.3) | Find the value of x so that matrix $\mathrm{A}=\left[\begin{array}{ccc}(x-1) & 1 & 1 \\ 1 & (x-1) & 1 \\ 1 & 1 & (x-1)\end{array}\right]$ is singular/ NonInvertible. |
| Sol.3) | Since matrix $A$ is singular $\begin{aligned} & \therefore\|A\|=0 \\ & \\ & \\ & \Rightarrow\left(\left.\begin{array}{ccc} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{array} \right\rvert\,=0\right. \\ & \Rightarrow(x-1)\left[(x-1)^{2}-1\right]-1[\mathrm{x}-1-1]+1[1-\mathrm{x}+1]=0 \\ & \Rightarrow(\mathrm{x}-1)\left(\mathrm{x}^{2}-2 \mathrm{x}\right)-1(\mathrm{x}-2)+(2-\mathrm{x})=0 \\ & \Rightarrow \mathrm{x}^{3}-2 \mathrm{x}^{2}-\mathrm{x}^{2}+2 \mathrm{x}-\mathrm{x}+2+2-\mathrm{x}=0 \\ & \Rightarrow \mathrm{x}^{3}-3 \mathrm{x}^{2}+4=0 \end{aligned}$ <br> By trial method $\begin{aligned} & (x+1)(x-2)(x+1)=0 \\ \Rightarrow & x=-1, x=2 \quad \text { ans. } \end{aligned}$ |
| Q.4) | (b) Prove that $\Delta=\left\|\begin{array}{ccc}x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x\end{array}\right\|$ is independent of $\theta$. |
| Sol.4) | $\begin{aligned} & \text { (a) we have, } \Delta=\left\|\begin{array}{ccc} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{array}\right\| \\ & \Rightarrow \quad \Delta=1\left(1+\sin ^{2} \theta\right)-\sin \theta(-\sin \theta+\sin \theta)+1\left(\sin ^{2} \theta+1\right) \\ & \Rightarrow \quad \Delta=1+\sin ^{2} \theta+0+\sin ^{2} \theta+1 \\ & \Rightarrow \quad \Delta=2+2 \sin ^{2} \theta \end{aligned}$ <br> Now, we know $\begin{array}{rlr}  & -1 \leq \sin \theta \leq 1 \\ \Rightarrow & 0 \leq \sin ^{2} \theta \leq 1 & \\ \Rightarrow & 0 \leq 2 \sin ^{2} \theta \leq 2 & \text { (multiply by 2) } \\ \Rightarrow & 2 \leq 2+2 \sin ^{2} \theta \leq 4 & \text { (adding 2) } \\ \Rightarrow & 2 \leq \Delta \leq 4 & \text { (proved) } \\ \text { (b) } \Delta=x\left(-x^{2}-1\right)-\sin \theta(-x \sin \theta-\cos \theta)+\cos \theta(-\sin \theta+x \cos \theta) \\ \Delta & =x^{3}-x+x \sin ^{2} \theta+\sin \theta \cos \theta-\sin \theta \cos \theta+x \cos ^{2} \theta \end{array}$ |

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|  | $\Delta=-x^{3}-x+x\left(\sin ^{2} \theta+\cos ^{2} \theta\right)$ |
| :--- | :--- |
| $\Delta$ | $=-x^{3}-x+x(1)$ |
| $\Delta$ | $=-x^{3} \quad$ which is independent of $\theta$. |

## Short Questions

Q.5) $\quad$ Order $3 \times 3,|A|=5$. Find $|\operatorname{Adj} A|=$ ?

Sol.5) We have $n=3,|A|=5$
and $|\operatorname{Adj} A|=|A|^{n-1}$

$$
=(5)^{3-1}=25 \quad \text { ans. }
$$

Q.6) $\quad$ Order $3 \times 3,|\operatorname{Adj} A|=81$ find $|A|=$ ?

Sol.6) We have $\mathrm{n}=3$, $|\operatorname{Adj} \mathrm{A}|=81$

$$
\begin{aligned}
& \Rightarrow|\operatorname{Adj} \mathrm{A}|=|\mathrm{A}|^{\mathrm{n}-1} \\
& \Rightarrow 81=|\mathrm{A}|^{2} \\
& \Rightarrow \quad|\mathrm{~A}|= \pm 9 \quad \text { ans. }
\end{aligned}
$$

Q.7) $\quad$ Order $3 \times 3$; $|A|=3$ find $|4 A|=$ ?

Sol.7) We have $n=3,|A|=3$

$$
\begin{equation*}
|4 A|=4^{3}|A| \tag{n}
\end{equation*}
$$

$=64 \times 3$

$$
=192 \text { ans. }
$$

Q.8) $\quad$ Order $3 \times 3 ;|A|=5$ find $|2 \operatorname{Adj} A|=$ ?

Sol.8) $\quad|2 \operatorname{Adj} A|=2^{3}|\operatorname{Adj} A|=2^{3}|A|^{3-1}$

$$
=8(5)^{2}=200 \quad \text { ans. }
$$

Q.9) $\quad \operatorname{Order} 4 \times 4 ;|3 \operatorname{Adj} A|=243$ Find $|A|=$ ?

Sol.9) We have $|3 \operatorname{Adj} A|=3^{4}|\operatorname{Adj} A|$

$$
\begin{aligned}
& 243=3^{4}|A|^{4-1} \\
& 243=81|A|^{3} \\
& |A|^{3}=3 \\
& |A|=(3)^{1 / 3} \quad \text { ans. }
\end{aligned}
$$

Q.10) Order $4 \times 4 ;|A|=5$ find $\left|A^{\prime}\right|=$ ?

Sol.10) We know $\left|A^{\prime}\right|=|A|$

$$
\Rightarrow\left|A^{\prime}\right|=5 \text { ans. }
$$

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