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# Determinants Class 12<sup>th</sup>

Q.1)	Find the value of x if the area of $\Delta$ is 35 square units with vertices $(x, 4), (2, -6)$ and $(5, 4)$ .
Sol.1)	Let vertices are $A(x,4)$ , $B(2,-6)$ and $C(5,4)$
	Area of $\triangle ABC = \frac{1}{2} \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$
	IJ + II
	$35 = \frac{1}{2}  x(-10) - 4(-3) + 1(38) $
	$\Rightarrow 35 = \frac{1}{2} \left  -10x + 12 + 38 \right $
	$\Rightarrow 70 =  -10x + 50 $
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
	$ \begin{vmatrix} 10x = -20 \\ x = -2 \end{vmatrix}                                 $
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
Q.3)	
	Find the value of x so that matrix A = $\begin{bmatrix} (x-1) & 1 & 1 \\ 1 & (x-1) & 1 \\ 1 & 1 & (x-1) \end{bmatrix}$ is singular/ Non-
6 10)	Invertible.
Sol.3)	Since matrix A is singular  ∴  A  = 0
	x - 1
	$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
	$\Rightarrow (x-1)[(x-1)^2-1]-1[x-1-1]+1[1-x+1]=0$ \Rightarrow (x-1)(x^2-2x)-1(x-2)+(2-x)=0
	$\Rightarrow x^3 - 2x^2 - x^2 + 2x - x + 2 + 2 - x = 0$
	$\Rightarrow x^3 - 3x^2 + 4 = 0$
	By trial method
	(x + 1) (x - 2) (x + 1) = 0
	$\rightarrow x = -1, x = 2$ difs.
Q.4)	(a) Evaluate the determinant $\Delta = \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \end{vmatrix}$ . Also prove $2 \le \Delta \le 4$ .
	$\begin{vmatrix} -1 & -\sin\theta & 1 \end{vmatrix}$
	$  x \sin \theta \cos \theta  $
	(b) Prove that $\Delta = \begin{vmatrix} -\sin \theta & -x & 1 \\ \cos \theta & 1 & y \end{vmatrix}$ is independent of $\theta$ .
Sol.4)	$\begin{vmatrix} \cos \theta & 1 & x \\ 1 & \sin \theta & 1 \end{vmatrix}$
	(a) we have, $\Delta = \begin{vmatrix} -\sin \theta & 1 & \sin \theta \end{vmatrix}$
	$\begin{vmatrix} -1 & -\sin \theta & 1 \\ \Rightarrow \Delta = 1(1 + \sin^2 \theta) - \sin \theta (-\sin \theta + \sin \theta) + 1(\sin^2 \theta + 1) \end{vmatrix}$
	$\Rightarrow \Delta = 1(1 + \sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$ $\Rightarrow \Delta = 1 + \sin^2\theta + 0 + \sin^2\theta + 1$
	$\Rightarrow \Delta = 1 + 3in \theta + \theta + 3in \theta + 1$ $\Rightarrow \Delta = 2 + 2 \sin^2 \theta$
	Now , we know
	$-1 \le \sin \theta \le 1$
	$\Rightarrow 0 \le \sin^2 \theta \le 1$
	$\Rightarrow 0 \le 2 \sin^2 \theta \le 2 \qquad \text{(multiply by 2)}$
	$\Rightarrow 2 \le 2 + 2\sin^2\theta \le 4 \qquad \text{(adding 2)}$
	$\Rightarrow 2 \le \Delta \le 4 $ (proved) (b) $\Delta = x(-x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta)$
	$\Delta = x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$ $\Delta = -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

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\Delta = -x^3 - x + x(\sin^2\theta + \cos^2\theta)
\Delta = -x^3 - x + x(1)
\Delta = -x^3 \quad \text{which is independent of } \theta.
```

#### **Short Questions**

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Q.5)
          Order 3 \times 3, |A| = 5. Find |Adj A| = ?
Sol.5)
          We have n = 3, |A| = 5
          and |Adj A| = |A|^{n-1}
                        =(5)^{3-1}=25 ans.
Q.6)
          Order 3 \times 3, |Adj A| = 81 find |A| = ?
          We have n = 3, |Adj A| = 81
Sol.6)
                    \Rightarrow |Adj A| = |A|^{n-1}
                    \Rightarrow 81 = |A|<sup>2</sup>
                    \Rightarrow |A| = \pm 9
                                       ans.
                                                               9.9.4.0014
Q.7)
          Order 3 \times 3; |A| = 3 find |4A| = ?
Sol.7)
          We have n = 3, |A| = 3
                                              \dots \{ : |kA| = k^n |A| \}
                     |4A| = 4^3 |A|
                         = 64 \times 3
                         = 192
                                   ans.
Q.8)
          Order 3 \times 3 ; |A| = 5 find |2Adj A| = ?
             |2Adj A| = 2^3 |Adj A| = 2^3 |A|^{3-1}
Sol.8)
                       = 8 (5)^2 = 200
Q.9)
          Order 4 \times 4; |3 \text{ Adj A}| = 243 \text{ Find } |A| = ?
Sol.9)
          We have |3 \text{ Adj A}| = 3^4 |\text{Adj A}|
                             243 = 3^4 |A|^{4-1}
                             243 = 81 |A|^3
                              |A|^3 = 3
                             |A| = (3)^{1/3}
Q.10)
          Order 4 \times 4; |A| = 5 find |A'| = ?
Sol.10)
          We know |A'| = |A|
                   \Rightarrow |A'| = 5 ans.
```

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