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Determinants

Class 12th

Q.1) Show that $\begin{vmatrix} 1 & 1 & 1 \\ mc_1 & m+1c_1 & m+2c_1 \\ mc_2 & m+1c_2 & m+2c_2 \end{vmatrix} = 1$

Sol.1) We have $\begin{vmatrix} 1 & 1 & 1 \\ mc_1 & m+1c_1 & m+2c_1 \\ mc_2 & m+1c_2 & m+2c_2 \end{vmatrix}$

= $\begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ \frac{m(m-1)}{2} & \frac{(m+1)m}{m} & \frac{(m+2)(m+1)}{2} \end{vmatrix}$ ..... $\left\{\because \begin{matrix} nc_1 = n \\ nc_2 = \frac{n(n-1)}{2} \end{matrix}\right\}$

taking $\left(\frac{1}{2}\right)$ common from R_3

= $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ m^2-m & m^2+m & m^2+3\mathrm{m}+2 \end{vmatrix}$

$c_2 \to c_2 - c_1 \text{and} c_3 \to c_3 - c_2$

= $\frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ m & 1 & 2 \\ m^2-m & 2\mathrm{m} & 4\mathrm{m}+2 \end{vmatrix}$

expanding along R_1

= $\frac{1}{2}[4\mathrm{m}+2-4\mathrm{m}]$

= $\frac{2}{2} = 1$ = RHS ans.

Q.2) Show $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2\mathrm{abc}(a+b+c)^3$

Sol.2) We have $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix}$

= $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$

= $\frac{1}{abc}\begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$

taking a , b , c common from $c_1, c_2 \text{and} c_3$ resp.

= $\frac{1}{abc}\begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$

$c_1 \to c_1 - c_3 \text{and} c_2 \to c_2 - c_3$

= $\begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (c+a+b)(c-a-b) & b^2 \\ (c+a+b)(c-a-b) & (c+a+b)(c-a-b) & (a+b)^2 \end{vmatrix}$

taking $(a+b+c)$ common from C_1 & C_2 both

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$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

$R_3 \to R_3(R_1 + R_2)$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2\text{b} & -2\text{a} & 2\text{ab} \end{vmatrix}$$

$c_1 \to ac_1 \text{and} c_2 \to bc_2$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ 0 & bc+ab-b^2 & b^2 \\ -2\text{ab} & -2\text{ab} & 2\text{ab} \end{vmatrix}$$

$c_1 \to c_1 + c_3 \text{and} c_2 \to c_2 + c_3$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac & a^2 & a^2 \\ b^2 & bc+ab & b^2 \\ 0 & 0 & 2\text{ab} \end{vmatrix}$$

taking a , b and 2ab common from R_1 , R_2 and R_3 resp.

$$= \frac{(a+b+c)^2}{ab}.ab(2\text{ab}) \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ 0 & 0 & 1 \end{vmatrix}$$

expanding

$= 2\text{ab}(a+b+c)^2[(b+c)(c+a) - a(b) + a(0)]$

$= 2\text{ab}(a+b+c)^2(bc+ab+c^2+ac-ab)$

$= 2\text{ab}(a+b+c)^2.c(b+c+a)$

$= 2\text{abc}(a+b+c)^3$ = RHS ans.

Q.3) Show that $\begin{vmatrix} 1+a^2-b^2 & 2\text{ab} & -2\text{b} \\ 2\text{ab} & 1-a^2+b^2 & 2\text{a} \\ 2\text{b} & -2\text{a} & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

Sol.3) We have $\begin{vmatrix} 1+a^2-b^2 & 2\text{ab} & -2\text{b} \\ 2\text{ab} & 1-a^2+b^2 & 2\text{a} \\ 2\text{b} & -2\text{a} & 1-a^2-b^2 \end{vmatrix}$

Main step $c_1 \to c_1 - bc_3 \text{and} c_2 \to ac_3$

$$= \begin{vmatrix} 1+a^2-b^2+2\text{b}^2 & 2\text{ab}-2\text{ab} & -2\text{b} \\ 2\text{ab}-2\text{ab} & 1-a^2+b^2+2\text{a}^2 & 2\text{a} \\ 2\text{b}-b+a^2b+b^3 & -2\text{a}+a-a^3-ab^2 & 1-a^2-b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2\text{b} \\ 0 & 1+a^2+b^2 & 2\text{a} \\ b(1+a^2+b^2) & -a(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}$$

taking $(1 + a^2 + b^2)$ common from $c_1 \text{and} c_2$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2\text{b} \\ 0 & 1 & 2\text{a} \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

expanding

$= (1+a^2+b^2)^2[1[1-a^2-b^2+2\text{a}^2] - 2\text{b}(-b)]$

$= (1+a^2+b^2)^2[1-a^2-b^2+2\text{a}^2+2\text{b}^2]$

$= (1+a^2+b^2)^2(1+a^2+b^2)$

$= (1+a^2+b^2)^3$ = RHS ans.

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Solving System of Linear Equations (Matrix Method)

Q.4) Solve the equations using matrix method $x + 2y + z = 7$; $x + 3z = 11$; $2x - 3y = 1$

Sol.4) The given equation are

$x + 2y + z = 7$
$x + 0y + 3z = 11$
$2x - 3y + 0z = 1$

these equation can be written in matrix form

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

(or) A x = B

$\Rightarrow$ x = A^{-1} B

Where $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$; $B \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$ & $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now

$|A| = 1(0+9) - 2(0-6) + 1(-3-0) = 9 + 12 - 3$

$|A| = 18 \neq 0$

$\therefore$ system is consistent and unique solution

Cofactors

$c_{11} = 9$; $c_{12} = -6$; $c_{14} = -3$
$c_{21} = -3$; $c_{22} = -2$; $c_{23} = 7$
$c_{31} = 6$; $c_{32} = -2$; $c_{33} = -2$

Now $Adj(A) = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}^7 = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} . \text{AoyA}$

$A^{-1} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$

We have x = A^{-1} B

$$x = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18}\begin{bmatrix} 63 - 33 + 6 \\ 42 - 22 - 2 \\ -21 + 77 - 2 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18}\begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore x = 2$, $y = 1$, $z = 3$ is the required solution ans.

Q.5) Solve the equations

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10 \ ; \ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10 \ ; \ \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

Sol.5) The given equations are

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$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$

$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$

These equations can be written in matrix form

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

(or) A X = B

$\Rightarrow$ x = A^{-1} B

Where $A = \begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$; $B = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$; $X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$

$|A| = 2(2+1) + 3(2-3) + 3(-1-3) = 6 - 3 - 12 = -9$

$|A| = -9 \neq 0$ $\therefore$ system is consistent and unique solution

Cofactors

$c_{11} = 3$ $c_{12} = 1$ $c_{13} = -4$

$c_{21} = 3$ $c_{22} = -5$ $c_{23} = -7$

$c_{31} = -6$ $c_{32} = 1$ $c_{33} = 5$

$\therefore$ AdjA = $\begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$

$A^2 = \frac{1}{|A|} \text{AdjA} = -\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$

We have x A^{-1} B

X = $-\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$

X = $-\frac{1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$

$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

$\Rightarrow$ x = $\frac{1}{2}$; y = $\frac{1}{3}$ and z = $\frac{1}{3}$ is the required solution Ans……

Q.6) Find A^{-1} , where A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the system of equations $x + 2y - 3z = -4$, $2x + 3y + 2z = 2$ and $3x - 3y - 4z = 11$

Sol.6) We have , A = $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$|A| = 1(-12+6) - 2(-8-6) - 3(-6-9) = -6 + 28 + 45$

$|A| = 67 \neq 0$ $\therefore$ (A is Invertible |consistent| unique solution)

Cofactors

$c_{11} = -6$ $c_{12} = 14$ $c_{13} = -15$

$c_{21} = 17$ $c_{22} = 5$ $c_{23} = 9$

$c_{31} = 13$ $c_{32} = -8$ $c_{33} = -1$

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$\therefore$ $Adj(A) = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

Now $A^{-1} = \frac{1}{|A|} . AdjA$

$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ ....(1)

Given equation are

$x + y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 1$

These equation can be written in the form

$\begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

(or) A X = B $\Rightarrow$ X = A^{-1} B

Where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$X = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$ ....{A^{-1} from eq. (i)}

$X = \frac{1}{67}\begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

$\therefore$ x = 3 , y = −2 , z = 1 is the required solution ans.

Q.7) If $= \begin{bmatrix} 1 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$. Find A–1 and hence solve the equation $x + 2y + z = 4$; $-x + y + z = 0$ and $x - 3y + z = 2$.

Sol.7) We have A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

Do yourself

$|A| = 10 \neq 0$ $\therefore$ (A is invertible)

$AdjA = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} . AdjA$

$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ .....(1)

Given equation are

$x + 2y + z = 4$
$-x + y + z = 0$
$x - 3y + z = 2$

$\rightarrow$ the matrix of above equation is clearly the transpose of given matrix A

$\therefore$ these equations can be written in the form

A' X = B where B = $\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\Rightarrow$ X = $(A^{-1})^{-1}$B

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$\Rightarrow$ X = $(A^{-1})^{-1}$B ....{By prop. $(A^{-1})^{-1} = (A^{-1})^{1}$}

$$\Rightarrow X = \frac{1}{10}\begin{bmatrix}4 & -5 & 1\\ 2 & 0 & -2\\ 2 & 5 & 3\end{bmatrix}\begin{bmatrix}4\\0\\2\end{bmatrix}$$

$$\Rightarrow X = \frac{1}{10}\begin{bmatrix}16+2\\ 8-4\\ 8+6\end{bmatrix} = \frac{1}{10}\begin{bmatrix}18\\4\\14\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}9/5\\2/5\\7/5\end{bmatrix}$$

$\Rightarrow x = \frac{9}{5}$, $y = \frac{2}{5}$, $z = \frac{7}{5}$ is the req. solution ans..

Q.8) Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and hence (or) use it to solve the equations $x - y + z = 4$; $x - 2y - 2z = 9$; $2x + y + 3z = 1$

Sol.8) Let $A = \begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and $C = \begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$

$$CA = \begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix} = \begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$$

$\Rightarrow CA = 8I$

Post multiply by A^{-1}

$\Rightarrow CAA^{-1} = 8I\,A^{-1}$

$\Rightarrow CI = 8A^{-1}$ ....$\left\{\begin{matrix}AA^{-1} = I\\ IA^{-1} = A^{-1}\end{matrix}\right\}$

$$\Rightarrow A^{-1} = \frac{1}{8}C = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$$

Given equation are

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

There equation can be in the form

$$\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & 2\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\9\\1\end{bmatrix}$$

(or) $AX = B$

$\Rightarrow X = A^{-1}B$ where B = $\begin{bmatrix}4\\9\\1\end{bmatrix}$; X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$

$$\Rightarrow X = \frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix}$$

$$\Rightarrow X = \frac{1}{8}\begin{bmatrix}24\\-16\\-8\end{bmatrix} = \begin{bmatrix}3\\-2\\-1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\-2\\-1\end{bmatrix}$$

$\Rightarrow x = 3, y = -2$ and $z = -1$ is the req. solution ans.

Q.9) A = $\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$ and B = $\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$ find AB and hence solve the equations $x - 2y = 0$; $2x + y + 3z = 8$; $-2y + x = 7$

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Sol.9) $AB = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$\Rightarrow AB = 11\,I$

Pre by A^{-1}

$\Rightarrow A^{-1}AB = 11\,A^{-1}I$

$\Rightarrow I\,B = 11\,A^{-1}$

$\Rightarrow B = 11\,A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{11}B1 = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

Given equations are

x – 2y = 10

2x + y + 3z = 7

–2y + 0y + z = 7

These equations can be written in the form

$A\,X = C \quad \Rightarrow \quad X = A^{-1}\,C$

Where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$X = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$\Rightarrow X = \frac{1}{11}\begin{bmatrix} 70+16-42 \\ -20+8-21 \\ -40+16+35 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$

$\therefore\ x = 4\ ,\ y = -3\ ,\ z = 1$ is the required solution ans.

Q.10) Show that system of equations is consistent and also find the solution

$2x - y + 3z = 5\ ;\ 3x + 2y - z = 7\ ;\ 4x + 5y - 5z = 9$

Sol.10) Given equation are

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y - 5z = 9$

given equation can be written in the form

$A\,X = B \quad \Rightarrow X = A^{-1}B$

where $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & -5 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$

|A| = 0 {solution can be infinite many or no solution}

Now $AdjA = \begin{bmatrix} -5 & 10 & -5 \\ 11 & -22 & 11 \\ 7 & -14 & 7 \end{bmatrix}$

Now $(AdjA)B = \begin{bmatrix} -5 & 10 & -5 \\ 11 & -12 & 11 \\ 7 & -14 & 7 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$

Since |A| = 0 also (AdjA). B = 0

$\therefore$ System is consistent and Infinite many solutions

→ Put $z = k$ in first two equations, we get

$2x - y = 5 - 3k \qquad \ldots..(k \in R)$

$3x + 2y = 7 + k$

(or) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5-3k \\ 7+k \end{bmatrix}$

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$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \; ; \; X = \begin{bmatrix} x \\ y \end{bmatrix} \; ; \; B = \begin{bmatrix} 5-3k \\ 7+k \end{bmatrix}$$

$|A| = 7$ and $AdjA = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 5-3k \\ 7+k \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 17-5k \\ 11k-1 \end{bmatrix}$$

$\therefore \; x = \frac{17-5k}{7} \; ; \; y = \frac{11k-1}{7}$ and $z = k$ ans.

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