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TOPIC 4
CONTINUITY AND DIFFRENTIABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
Continuity& Differentiability	1.Limit of a function		
	2.Continuity	***	Ex 5.1 Q.No- 21, 26,30
	3.Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
	4.Logrithmic Differentiation	***	Ex 5.5 QNo- 6,9,10,15
	5 Parametric Differentiation	***	Ex 5.6 QNo- 7,8,10,11
	6. Second order derivatives	***	Ex 5.7 QNo- 14,16,17
	7. Mean Value Theorem	**	Ex 5.8 QNo- 3,4

SOME IMPORTANT RESULTS/CONCEPTS

* A function f is said to be continuous at x = a if Left hand limit = Right hand limit = value of the function at x = a

i.e. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$

i.e. $\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a-h) = f(a)$.

* A function is said to be differentiable at x = a if $Lf'(a) = Rf'(a)$ i.e

$$\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

(i) $\frac{d}{dx}(x^n) = n\,x^{n-1}$.

(ii) $\frac{d}{dx}(x) = 1$

(iii) $\frac{d}{dx}(c) = 0, \forall\, c \in R$

(iv) $\frac{d}{dx}(a^x) = a^x \log a,\ a > 0,\ a \neq 1$.

(v) $\frac{d}{dx}(e^x) = e^x$.

(vi) $\frac{d}{dx}(\log_a x) =, \frac{1}{x\log a}\ a > 0,\ a \neq 1,\ x$

(vii) $\frac{d}{dx}(\log x) = \frac{1}{x},\ x > 0$

(xiii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x,\ \forall\, x \in R$.

(xiv) $\frac{d}{dx}(\sec x) = \sec x \tan x,\ \forall\, x \in R$.

(xv) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x,\ \forall\, x \in R$.

(xvi) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$.

(xvii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$.

(xviii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2},\ \forall\ x \in R$

(xix) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2},\ \forall\, x \in R$.

(xx) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$,.

(xxi) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$.

(xxii) $\frac{d}{dx}(|x|) = \frac{x}{|x|},\ x \neq 0$

(xxiii) $\frac{d}{dx}(ku) = k\frac{du}{dx}$

(xxiv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

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(viii) $\frac{d}{dx}(\log_a\lvert x \rvert) = \frac{1}{x \log a}$, $a > 0, a \neq 1, x \neq 0$ (ix) $\frac{d}{dx}(\log\lvert x \rvert) = \frac{1}{x}, x \neq 0$ (x) $\frac{d}{dx}(\sin x) = \cos x, \forall x \in R.$ (xi) $\frac{d}{dx}(\cos x) = -\sin x, \forall x \in R.$ (xii) $\frac{d}{dx}(\tan x) = \sec^2 x, \forall x \in R.$	(xxv) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$ (xxvi) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

2.Continuity

LEVEL-I

1.Examine the continuity of the function $f(x)=x^2 + 5$ at x=-1.

2. Examine the continuity of the function $f(x)=\frac{1}{x+3}, x \in R.$

3. Show that f(x)=4x is a continuous for all $x \in R.$

LEVEL-II

1. Give an example of a function which is continuous at x=1,but not differentiable at x=1.

2. For what value of k,the function $\begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at x=2.

3.Find the relationship between "a" and "b" so that the function 'f' defined by:

[CBSE 2011]

$f(x)=\begin{cases} ax+1 & \text{if } x \leq 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at x=3.

4. If $f(x)=\begin{cases} \frac{\sin 3x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$. Find whether f(x) is continuous at x=0.

LEVEL-III

1.For what value of k, the function $f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x=0?

2. If function $f(x)=\frac{2x+3\sin x}{3x+2\sin x}$, for $x \neq 0$ is continuous at x=0, then Find f(0).

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3.Let $f(x)\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2}\\ a & \text{if } x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & \text{if } x>\dfrac{\pi}{2}\end{cases}$ = If f(x) be a continuous function at x= $\dfrac{\pi}{2}$, find a and b.

4.For what value of k,is the function f(x) = $\begin{cases}\dfrac{\sin x + x\cos x}{x} & \text{, when } x\neq 0\\ k & \text{, when } x=0\end{cases}$ continuous at x= 0?

3.Differentiation

LEVEL-I

1. Discuss the differentiability of the function $f(x)=(x-1)^{2/3}$ at x=1.

2.Differentiate $y=\tan^{-1}\dfrac{2x}{1-x^2}$.

3. If $y=\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$, Find $\dfrac{dy}{dx}$.

LEVEL-II

1. Find $\dfrac{dy}{dx}$, $y = \cos(\log x)^2$.

2. Find $\dfrac{dy}{dx}$ of $y=\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$

3.If $y=e^{ax}\sin bx$, then prove that $\dfrac{d^2y}{dx^2}-2a\dfrac{dy}{dx}+(a^2+b^2)y=0$.

4.Find $\dfrac{d^2y}{dx^2}$, if $y=\dfrac{3at}{1+t}$, $x=\dfrac{2at^2}{1+t}$.

LEVEL-III

1.Find $\dfrac{dy}{dx}$, if $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

2.Find $\dfrac{dy}{dx}$ $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, $0<x<\dfrac{\pi}{2}$.

3. If $y=\sin^{-1}\left(\dfrac{a+b\cos x}{b+a\cos x}\right)$, show that $\dfrac{dy}{dx}=\dfrac{-\sqrt{b^2-a^2}}{b+a\cos x}$.

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4.Prove that $\frac{d}{dx}\left[\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right|+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}\,x}{1-x^2}\right)\right]=\frac{1}{1+x^4}$.

4.Logrithmic Differentiation

LEVEL-I

1.Differentiate $y=\log_7(\log x)$.

2. Differentiate , sin(log x),with respect to x.

3.Differentiate $y=\tan^{-1}(\log x)$

LEVEL-II

1. If $y.\sqrt{x^2+1}=\log[\sqrt{x^2+1}-x]$,show that $(x^2+1)\frac{dy}{dx}+xy+1=0$.
2. Find $\frac{dy}{dx}$, $y = \cos(\log x)^2$.
3. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$ **[CBSE 2012]**

LEVEL-III

1.If $x^p.y^q=(x+y)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$

2. $y=(\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$

3. If $x^y = e^{x-y}$, Show that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$ **[CBSE 2011]**

4. Find $\frac{dy}{dx}$ when $y=x^{\cot x}+\frac{2x^2-3}{x^2+x+2}$ **[CBSE 2012]**

5 Parametric Differentiation

LEVEL-II

1.If $y = \tan x$, prove that $\frac{d^2y}{dx^2}=2y\frac{dy}{dx}$

2. .If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$ find $\frac{d^2y}{dx^2}$at $\theta=\frac{\pi}{4}$.

3. If $x=\tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-a)=0$ **[CBSE 2011]**

6. Second order derivatives

LEVEL-II

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1. If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

2.If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$

3.If $(x-a)^2+(x-b)^2=c^2$ for some c>0.Prove that $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant, independent

7. Mean Value Theorem

LEVEL-II

1.It is given that for the function $f(x)=x^3-6x^2+px+q$ on[1,3] , Rolle's theorem holds with

$c=2+\frac{1}{\sqrt{3}}$. Find the values p and q.

2. Verify Rolle's theorem for the function f(x) = sinx, in $[0,\pi]$.Find c, if verified

3.VeifyLagrange's mean Value Theorem $f(x)=\sqrt{x^2-4}$ in the interval [2,4]

Questions for self evaluation

1.For what value of k is the following function continuous at x = 2 ?

$$f(x)=\begin{cases}2x+1\,;\,x<2\\ k\,;\,x=2\\ 3x-1\,;\,x>2\end{cases}$$

2.If $f(x)=\begin{cases}3ax+b, & \text{if } x>1\\ 11 & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$, continuous at x = 1, find the values of a and b.**[CBSE 2012 Comptt.]**

3. Discuss the continuity of $f(x)=|x-1|+|x-2|$ at x = 1 & x = 2.

4. If f(x), defined by the following is continuous at x = 0, find the values of a, b, c

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x} & ,x<0\\ c & ,x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & ,x>0\end{cases}$$

5.If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$ find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$.

6. If $y=(\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$.

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7. If $xy + y^2 = \tan x + y$, find $\frac{dy}{dx}$.

8. If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.

9. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

10. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$

11. If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

12. If $x^p . y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

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