CHAPTER - 5

CONTINUITY AND DIFFERENTIATION

I. Derivative of an exponential function

Let $f(x) = e^x$

Then by definition

$$f^1(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} e^x \left(\frac{e^h - 1}{h}\right)$$

$$= e^x \lim_{h \to 0} \frac{(e^h - 1)}{h} = e^x . 1 = e^x$$

Hence, $\frac{d}{dx}(e^x) = e^x$

Similarly, we can write that $\frac{d}{dx}(a^x) = a^x \log a$.

II. Derivative of a logarithmic function

Let $f(x) = \log_e x$, where $x > 0$

Then, $f^1(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\log_e(x+h) - \log_e x}{h}$$

$$= \lim_{h \to 0} \frac{\log_e\left(\frac{x+h}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\log_e\left(1+\frac{h}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\log_e\left(1+\frac{h}{x}\right)}{\frac{h}{x}} \frac{1}{x} = 1.\frac{1}{x} = \frac{1}{x}$$

Hence, $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

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