

### **Chapter: -Continuity and Differentiability**

#### **1 marks question**

- Q1.** Find  $\frac{dy}{dx}$  of following functions:- (i)  $y = \log(\sin x)$  (ii)  $y = \sqrt{e^{\sqrt{x}}}$  (iii)  $y = \log_{10} x$  (iv)  $y = e^{2x+3}$   
 (v)  $y = \tan^{-1} \sqrt{x}$  (vi)  $y = 2^{\sin x}$  (vii)  $y = \log(\log x)$  (viii)  $y = 3 \sin^{-1} 2x$  (ix)  $y = \sqrt{3x+5}$  (x)  $y = x^x$   
 (xi)  $y = \sin^{-1}(2x\sqrt{1-x^2})$  (xii)  $y = 2^{\cos^2 x}$  (xiii)  $y = \cos(\tan \sqrt{x+1})$  (xiv)  $y = \sin \sqrt{x} + \cos^2 \sqrt{x}$   
 (xv)  $y = \sin^n(ax^2 + bx + c)$  (xvi)  $y = \log[\log(\log x^5)]$  (xvii)  $y = \sin x^2 + \sin^2 x + \sin^2(x^2)$

**Ans.** (i).  $\cot x$  . (ii)  $\frac{\sqrt{e^{\sqrt{x}}}}{\sqrt{x}}$  (iii)  $\frac{1}{x} \log_{10} e$  . (iv)  $2e^{2x+3}$  . (v)  $\frac{1}{2\sqrt{x(1+x)}}$  . (vi)  $2^{\sin x} \cos x \log 2$  .  
 (vii)  $\frac{1}{x \log x}$  (viii)  $\frac{6}{\sqrt{1-4x^2}}$  . (ix)  $\frac{3}{2\sqrt{3x+5}}$  . (x)  $x^x(1+\log x)$  . (xi)  $\frac{2}{\sqrt{1-x^2}}$  (xii)  $-2^{\cos^2 x} (\log 2x) \sin 2x$   
 (xiii)  $\frac{1}{2\sqrt{x+1}} \sin(\tan \sqrt{x+1}) \sec^2(\sqrt{x+1})$  (xiv)  $\frac{\cos \sqrt{x}}{2\sqrt{x}} - \frac{\sin 2\sqrt{x}}{2\sqrt{x}}$   
 (xv)  $n(2ax+b) \sin^{n-1}(ax^2 + bx + c) \cos(ax^2 + bx + c)$  (xvi)  $\frac{5}{x \log(x^5)[\log(\log x^5)]}$   
 (xvii)  $2x \cos x^2 + \sin 2x + 2x \sin(2x^2)$

**Q2.** Discuss the differentiability of  $f(x) = |x| \forall x \in \mathbb{R}$ . **Ans.** Not differentiable at  $x=0$ .

**Q3.** Discuss the differentiability of  $f(x) = [x]$ , where  $[x]$  is greatest integer function ,at  $x=2$  and  $5/2$ .

**Ans.** Not differentiable at  $x=2$  and differentiable at  $x=5/2$ .

#### **4/6 marks question**

**Q4.** If  $f(x) = \begin{cases} \frac{5x+|x|}{3x} & \text{if } x \neq 0 \\ \frac{2}{2} & \text{if } x = 0 \end{cases}$ , Show that  $f(x)$  is discontinuous at  $x=0$ .

**Q5.** Show that function  $f$ , defined by  $f(x) = \begin{cases} 2x & \text{if } x < 2 \\ 2 & \text{if } x = 2 \\ x^2 & \text{if } x > 2 \end{cases}$ , is discontinuous at  $x=2$ .

**Q6.** Find the value of  $k$  if  $f(x) = \begin{cases} \frac{\sin 5x}{3x} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ , is continuous at  $x=0$ . **Ans.** 5/3

**Q7.** Check the continuity of  $f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$ , **Ans.** Continuous. P.T.O.

**Q8.** Find  $\frac{dy}{dx}$  of following functions :- (i)  $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$  (ii)  $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$  . (iii)  $y = \log \sqrt{\frac{1+\sin x}{1-\sin x}}$

(iv)  $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$  (v)  $y = \log\left(\sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}\right)$  (vi)  $y = 5^{\log(\sin x)} + \sin^x x$  (vii)  $y = (\log x)^x + x^{\log x}$

(viii)  $y = \log\left(\frac{x+\sqrt{x^2+a^2}}{x-\sqrt{x^2-a^2}}\right)$  (ix)  $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$  (x)  $y = (x^x)^x$

**Ans.** (i)  $\frac{1}{2}$  . (ii)  $\frac{1}{2} \sec^2 \frac{x}{2}$  (iii)  $\sec x$  (iv)  $\cdot \frac{1}{2(1+x^2)}$  (v)  $\frac{1}{2} \left( \frac{-\sin 2x}{1+\cos^2 2x} + \frac{2e^{2x}}{1-e^{2x}} \right)$

(vi)  $\cot x 5^{\log(\sin x)} \log 5 + \sin^x x (x \cot x + \log \sin x)$  (vii)  $(\log x)^x \left( \frac{1}{\log x} + \log x (\log x) \right) + \frac{2 \log x}{x} x^{\log x}$

(viii)  $\left( \frac{1}{\sqrt{x^2+a^2}} + \frac{1}{\sqrt{x^2-a^2}} \right)$  (ix)  $\left( \frac{x}{\sqrt{1-x^4}} \right)$  (x)  $(x^x)^x (x+2x \log x)$

**Q9.** (i) If  $y = \left( \frac{\sin^{-1} x}{\sqrt{1-x^2}} \right)$  then prove that  $(1-x^2) \frac{dy}{dx} - xy = 1$ .

(ii) If  $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$  then prove that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

(iii) If  $y = \sqrt{\sin x + \sqrt{\sin x + \dots}}$  then prove that  $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

(iv) If  $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$  then prove that  $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$

(v) If  $x = a \left( \cos t + \log\left(\tan \frac{t}{2}\right) \right)$ ,  $y = a \sin t$ , then show that  $\frac{dy}{dx} = 1$  at  $t = \frac{\pi}{4}$

(vi) If  $y = x \log \frac{x}{a+bx}$ , then show that  $x^3 \frac{d^2 y}{d^2 x} = \left( x \frac{dy}{dx} - y \right)^2$

**Q10.** Find  $\frac{d^2 y}{d^2 x}$  if (i)  $y = \frac{3at}{1+t}$ ,  $x = \frac{2at^3}{1+t}$  (ii)  $y = a(1-\cos \theta)$ ,  $x = a(\theta - \sin \theta)$ , at  $\theta = \frac{\pi}{2}$

(iii)  $y = 2at^2$ ,  $x = t^3$  **Ans.** (i)  $\frac{-3t(1+t)^2}{2(1+t^2)^3}$  . (ii)  $\frac{-1}{a}$  (iii)  $\frac{-4a}{9t^4}$

**Q11.** Verify **Roll's Theorem** if (i)  $f(x) = \cos 2x$  in  $0 \leq x \leq 2\pi$ , (ii)  $f(x) = (x-a)^3(x-b)^4$  in  $(a,b)$

**Ans.** (i)  $c = \pi/2$ , (ii)  $c = (4a+3b)/7$ .

**Q12.** Verify **LMV Theorem** if  $f(x) = \sqrt{25-x^2}$  in  $[-3, 4]$  **Ans.**  $c = \pm \frac{1}{\sqrt{2}}$

-----Best of Luck-----