

**Class XII Assignment On Continuity And Differentiability**

Q.1. If  $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ a, & \text{if } x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2}, & \text{if } x > \frac{\pi}{2} \end{cases}$  is continuous function at  $x = \frac{\pi}{2}$ , find a and b.

Q.2. If  $y = (\sin x)^x + \sin^{-1} x - 2^{\sin x}$ , find  $\frac{dy}{dx}$

Q.3. If  $x = a(\theta - \sin \theta)$ ,  $y = a(1 + \cos \theta)$ , find  $\frac{d^2 y}{dx^2}$  at  $\theta = \frac{\pi}{2}$ .

Q.4. If  $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & \text{if } x > 0 \end{cases}$ , is continuous at  $x = 0$ , determine the value of a.

Q.5. Prove that the function  $f$  given by  $f(x) = |x - 3|$ ,  $x \in \mathbb{R}$  is continuous but not differentiable at  $x = 3$ .

Q.6. If  $x = \cos t + \log \tan \frac{t}{2}$  and  $y = \sin t$  then find  $\frac{d^2 y}{dx^2}$  at  $t = \frac{\pi}{4}$ .

Q.7. Find  $\frac{dy}{dx}$  when  $y = (\sin x)^{\cos x} + \frac{2x}{x^2 + x + 2}$ .

Q.8. If  $(\sin x)^{\cos y} = (\cos y)^{\sin x}$ , find the value of  $dy/dx$ .

Q.9. If  $y = a \sin x + b \cos x$ , prove that  $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$ .

Q.10. Prove that  $\frac{d}{dx} \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) \right] = \sqrt{a^2 - x^2}$ .

Q.11. If  $x = \tan \left( \frac{1}{a} \log y \right)$ , Show that  $(1+x^2) \frac{d^2 y}{dx^2} + (2x-a) \frac{dy}{dx} = 0$ .

Q.12. If  $y = \cos^{-1} \left( \frac{3x + 4\sqrt{1-x^2}}{5} \right)$ , Find  $\frac{dy}{dx}$ .

Q.13. If  $y = \operatorname{cosec}^{-1} x$ ,  $x > 1$ , then show that  $x(x^2 - 1) \frac{d^2 y}{dx^2} + (2x^2 - 1) \frac{dy}{dx} = 0$ .

Q.14. Show that the function  $f$  defined as follows, is continuous at  $x = 2$  but not

differentiable there.  $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$

Q.15. Find  $\frac{dy}{dx}$  if  $y = \sin^{-1}(x^2 \sqrt{1-x^2} - x \sqrt{1-x^4})$