CBSE Class 12 Mathematics Differentiation Worksheet

Rolle's And Mean Value Theorem

Q1. If f ; $[-5,5] \rightarrow R$ is differentiable and if $f^{I}(x)$ does not vanish anywhere, then show that $f(-5) \neq f(5)$.

Sol.1 Here a = -5 and b = 5

let us assume that $f(-5) = f(5)$ ….{i.e f(a) = f(b)}

we are given ; $f(x)$ is differentiable function is continuous

∴ f(x) is also continuous

Hence the three conditions of Rolle's theorem are satisfied.

∴ there must exists a value

c ← (-5 , 5) such that $f^{I}(c) = 0$

But we are given that $f^{I}(x)$ does not vanish anywhere (i.e. $f^{I}(x) \neq 0$)

∴ our assumption in wrong

$\Rightarrow f(-5) \neq f(5)$ (Proved)

Miscellaneous – Type - Questions

Q2. If $y = x \log\left(\frac{x}{a+bx}\right)$, show that $x^3.\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

Sol.2
$$y = x \log\left(\frac{x}{a+bx}\right)$$

$$\Rightarrow y = x[\log x - \log(a+bx)]$$

Diff w.r.t. x (product rule)

$$\frac{dy}{dx} = x\left[\frac{1}{x} - \frac{b}{a+bx}\right] + (\log x - \log(a+bx)).1$$

$$\frac{dy}{dx} = x\left[\frac{a+bx-bx}{x(a+bx)}\right] + \log x - \log(a+bx)$$

$$\frac{dy}{dx} = \frac{a}{a+bx} + \log x - \log(a+bx)$$

Diff again

$$\frac{d^2y}{dx^2} = \frac{(a+bx)(0)-a(b)}{(a+bx)^2} + \frac{1}{x} - \frac{b}{a+bx}$$

$$\frac{d^2y}{dx^2} = \frac{-ab}{(a+bx)^2} + \frac{1}{x} - \frac{b}{a+bx}$$

$$\frac{d^2y}{dx^2} = \frac{-abx+(a+bx)^2-b(a+bx)x}{x(a+bx)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-abx+a^2+b^2x^2+2\text{abx}-abx-b^2x^2}{x(a+bx)^2}$$

$$\frac{d^2y}{dx^2} = \frac{a^2}{x(a+bx)^2}$$

LHS $x^3.\frac{d^2y}{dx2} = x^3.\frac{a^2}{x(a+bx)^2} = \frac{a^2x^2}{(a+bx)^2}$

Now RHS $\left(x.\frac{dy}{dx} - y\right)^2$

$= \left[x\left(\frac{a}{a+bx+\log x-\log(a+bx)}\right) - x\log\left(\frac{x}{a+bx}\right)\right]$

$= \left[\frac{ax}{a+bx} + x.\log\left(\frac{x}{a+bx}\right) - x\log\left(\frac{x}{a+bx}\right)\right]^2$

$= \left(\frac{ax}{a+bx}\right)^2 = \frac{a^2x^2}{(a+bx)^2}$= LHS Proved

Q3. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\infty$ then write $\frac{d^2y}{dx^2}$ in terms of y.

Sol.3 Diff w.r.t. x

$$\frac{dy}{dx} = -1 + \frac{2x}{2!} - \frac{3x^2}{3!} + \frac{4x^3}{4!} \ldots\infty$$

$$\frac{d^2y}{dx^2} = -1 + x - \frac{x^2}{2} + \frac{x^3}{6} \ldots\ldots\infty$$

$$\frac{d^2y}{dx^2} = 1 - \frac{2x}{2} + \frac{3x^2}{6} \ldots\ldots\infty$$

$$\frac{d^2y}{dx^2} = 1 - \frac{x}{1!} + \frac{x^2}{2!} \ldots\ldots\infty$$

$\frac{d^2y}{dx^2} = y$ Ans.

Q4. If $y = |\log x|$. Find $\frac{d^2y}{dx^2}$.

Sol.4 Now point log x < 0 when 0 < x < 1

log x > 0 when x > 1

$\therefore$ y = {–log x ; 0 < x < 1}

{ log x ; x > 1}

Diff w.r.t x

$$\frac{dy}{dx} = \left\{\frac{-1}{x}; 0 < x < 1\right\}$$

$\left\{\frac{1}{x}; x > 1\right\}$

Diff again

$\frac{d^2y}{dx^2} = \left\{\frac{1}{x^2}; 0 < x < 1\right\}$

$\left\{\frac{-1}{x^2}; x > 1\right\}$ Ans.

Q5. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f^1(x) = \sin(x^2)$. Find $\frac{dy}{dx}$.

Sol.5 $y = f\left(\frac{2x-1}{x^2+1}\right)$

Diff w.r.t. x $\frac{dy}{dx} = f^1\left(\frac{2x-1}{x^2+1}\right) \cdot \frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$

$\frac{dy}{dx} = \sin\left(\frac{2x-1}{x^2+1}\right)^2 \cdot \left[\frac{(x^2+1)(2)-(2x-1)(2x)}{(x^2+1)^2}\right]$

......$\left\{\text{since} f^1(x) = \sin(x^2), f^1\left(\frac{2x-1}{x^2+1}\right) = \sin\left(2x - \frac{1}{tx^2} + 1\right)^2\right\}$

$\frac{dy}{dx} = \sin\left(\frac{2x-1}{x^2+1}\right)^2 \cdot \left[\frac{2x^2+2-4x^2+2x}{(x^2+1)^2}\right]$

$\frac{dy}{dx} = \sin\left(\frac{2x-1}{x^2+1}\right)^2 \cdot \left(\frac{2+2x-2x^2}{(x^2+1)^2}\right)$ Ans.