## CBSE Class 12 Mathematics Differentiation

## Worksheet

## Rolle's And Mean Value Theorem

Q1. If $f ;[-5,5] \rightarrow R$ is differentiable and if $\mathrm{f}^{1}(\mathrm{x})$ does not vanish anywhere, then show that $f(-5) \neq f(5)$.

Sol. 1 Here $\mathrm{a}=-5$ and $\mathrm{b}=5$
let us assume that $f(-5)=f(5) \ldots$ \{i.e $\mathrm{f}(\mathrm{a})=\mathrm{f}(\mathrm{b})\}$
we are given ; $f(x)$ is differentiable function is continuous
$\therefore \mathrm{f}(\mathrm{x})$ is also continuous
Hence the three conditions of Rolle's theorem are satisfied.
$\therefore$ there must exists a value
$\mathrm{c} \leftarrow(-5,5)$ such that $\mathrm{f}^{1}(\mathrm{c})=0$
But we are given that $\mathrm{f}^{1}(\mathrm{x})$ does not vanish anywhere (i.e. $\mathrm{f}^{1}(x) \neq 0$ )
$\therefore$ our assumption in wrong
$\Rightarrow f(-5) \neq f(5) \quad$ (Proved)

## Miscellaneous - Type - Questions

Q2. If $y=x \log \left(\frac{x}{a+b x}\right)$, show that $x^{3} \cdot \frac{d^{2} y}{d x^{2}}=\left(x \frac{d y}{d x}-y\right)^{2}$.
Sol. 2

$$
\begin{aligned}
y & =x \log \left(\frac{x}{a+b x}\right) \\
\Rightarrow y= & x[\log x-\log (a+b x)]
\end{aligned}
$$

Diff w.r.t. © (product rule)

$$
\begin{aligned}
& \frac{d y}{d x}=x\left[\frac{1}{x}-\frac{b}{a+b x}\right]+(\log x-\log (a+b x)) \cdot 1 \\
& \frac{d y}{d x}=x\left[\frac{a+b x-b x}{x(a+b x)}\right]+\log x-\log (a+b x) \\
& \frac{d y}{d x}=\frac{a}{a+b x}+\log x-\log (a+b x)
\end{aligned}
$$

Diff again

$$
\begin{aligned}
& \frac{d^{2} y}{d x^{2}}=\frac{(a+b x)(0)-a(b)}{(a+b x)^{2}}+\frac{1}{x}-\frac{b}{a+b x} \\
& \frac{d^{2} y}{d x^{2}}=\frac{-a b}{(a+b x)^{2}}+\frac{1}{x}-\frac{b}{a+b x}
\end{aligned}
$$

Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission.

## Downloaded from www.studiestoday.com

## StudiesToday

$$
\begin{aligned}
& \frac{d^{2} y}{d x^{2}}=\frac{-a b x+(a+b x)^{2}-b(a+b x) x}{x(a+b x)^{2}} \\
& \frac{d^{2} y}{d x^{2}}=\frac{-a b x+a^{2}+b^{2} x^{2}+2 \mathrm{abx}-a b x-b^{2} x^{2}}{x(a+b x)^{2}} \\
& \frac{d^{2} y}{d x^{2}}=\frac{a^{2}}{x(a+b x)^{2}}
\end{aligned}
$$

LHS $x^{3} \cdot \frac{d^{2} y}{d x 2}=x^{3} \cdot \frac{a^{2}}{x(a+b x)^{2}}=\frac{a^{2} x^{2}}{(a+b x)^{2}}$
Now RHS $\left(x \cdot \frac{d y}{d x}-y\right)^{2}$

$$
\begin{aligned}
& =\left[x\left(\frac{a}{a+b x+\log x-\log (a+b x)}\right)-x \log \left(\frac{x}{a+b x}\right)\right] \\
& =\left[\frac{a x}{a+b x}+x \cdot \log \left(\frac{x}{a+b x}\right)-x \log \left(\frac{x}{a+b x}\right)\right]^{2} \\
& =\left(\frac{a x}{a+b x}\right)^{2}=\frac{a^{2} x^{2}}{(a+b x)^{2}}=\text { LHS Proved }
\end{aligned}
$$

Q3. If $y=1-x+\frac{x^{2}}{2!}-\frac{x^{3}}{3!}+\frac{x^{4}}{4!}+\ldots \ldots \infty$ then write $\frac{d^{2} y}{d x^{2}}$ in terms of $y$.
Sol. 3 Diff w.r.t. $x$

$$
\begin{aligned}
& \frac{d y}{d x}=-1+\frac{2 \mathrm{x}}{2!}-\frac{3 \mathrm{x}^{2}}{3!}+\frac{4 \mathrm{x}^{3}}{4!} \ldots \infty \\
& \frac{d^{2} y}{d x^{2}}=-1+x-\frac{x^{2}}{2}+\frac{x^{3}}{6} \ldots \infty \\
& \frac{d^{2} y}{d x^{2}}=1-\frac{2 \mathrm{x}}{2}+\frac{3 \mathrm{x}^{2}}{6}, \ldots \infty \\
& \frac{d^{2} y}{d x^{2}}=1-\frac{x}{1!}+\frac{x^{2}}{2!} \ldots \infty \\
& \frac{d^{2} y}{d x^{2}}=y \text { Ans. }
\end{aligned}
$$

Q4. If $y=|\log x|$. Find $\frac{d^{2} y}{d x^{2}}$.
Sol. 4 Now point $\log x<0$ when $0<x<1$

$$
\log x>0 \quad \text { when } \quad x>1
$$

$\therefore \quad y=\{-\log x ; 0<x<1\}$
$\{\log \mathrm{x} ; \mathrm{x}>1\}$
Diff w.r.t x

$$
\frac{d y}{d x}=\left\{\frac{-1}{x} ; 0<x<1\right\}
$$

Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission.

## Downloaded from www.studiestoday.com

## StudiesToday

$$
\left\{\frac{1}{x} ; x>1\right\}
$$

Diff again

$$
\begin{aligned}
\frac{d^{2} y}{d x^{2}}=\left\{\frac{1}{x^{2}} ; 0<x<1\right\} \\
\left\{\frac{-1}{x^{2}} ; x>1\right\} \quad \text { Ans. }
\end{aligned}
$$

Q5. If $y=f\left(\frac{2 \mathrm{x}-1}{x^{2}+1}\right)$ and $f^{1}(x)=\sin \left(x^{2}\right)$. Find $\frac{d y}{d x}$.
Sol. $5 \quad y=f\left(\frac{2 x-1}{x^{2}+1}\right)$
Diff w.r.t. $\mathrm{x} \frac{d y}{d x}=f^{1}\left(\frac{2 \mathrm{x}-1}{x^{2}+1}\right) \cdot \frac{d}{d x}\left(\frac{2 \mathrm{x}-1}{x^{2}+1}\right)$

$$
\begin{aligned}
\frac{d y}{d x}=\sin \left(\frac{2 \mathrm{x}-1}{x^{2}+1}\right)^{2} \cdot\left[\frac{\left(x^{2}+1\right)(2)-(2 \mathrm{x}-1)(2 \mathrm{x})}{\left(x^{2}+1\right)^{2}}\right] \\
\ldots \ldots\left\{\operatorname{since} f^{1}(x)=\sin \left(x^{2}\right), f^{1}\left(\frac{2 \mathrm{x}-1}{x^{2}+1}\right)=\sin \left(2 \mathrm{x}-\frac{1}{t x^{2}}+1\right)^{2}\right\} \\
\frac{d y}{d x}=\sin \left(\frac{2 \mathrm{x}-1}{x^{2}+1}\right)^{2} \cdot\left[\frac{2 \mathrm{x}^{2}+2-4 \mathrm{x}^{2}+2 \mathrm{x}}{\left(x^{2}+1\right)^{2}}\right] \\
\frac{d y}{d x}=\sin \left(\frac{2 \mathrm{x}-1}{x^{2}+1}\right)^{2} \cdot\left(\frac{2+2 \mathrm{x}-2 \mathrm{x}^{2}}{\left(x^{2}+1\right)^{2}}\right)
\end{aligned}
$$

Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including
photocopying, recording, or other electronic or mechanical methods, without the prior written permission.

