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## CBSE Class 12 Mathematics Differentiation Worksheet

## Continuity \& Differentiability

Q1. Find the value of ' a ' and ' b ' so that $\mathrm{f}(\mathrm{x})$ is continues at $\mathrm{x}=4$.

$$
\begin{aligned}
\mathrm{f}(\mathrm{x})= & \left\{\frac{x-4}{(x-4)}+a ; x<4\right\} \\
& \{a+b ; x=4\} \\
& \left\{\frac{x-4}{(x-4)}+b ; x>4\right\}
\end{aligned}
$$

Sol. 1 Redefining the given $\mathrm{f}(\mathrm{x})$

$$
\begin{aligned}
& \mathrm{f}(\mathrm{x})=\left\{\frac{(x-4)}{-(x-4)}+a ; x<4\right\} \\
&\{\mathrm{a}+\mathrm{b} \quad ; \mathrm{x}=4\} \\
&\left\{\frac{(x-4)}{(x-4)}+b ; x>4\right\}
\end{aligned}
$$

since $\mathrm{x}<4 \quad \therefore|\mathrm{x}-4|=-(\mathrm{x}-4)$
and $\quad x>4 \quad \therefore|x-4|=(x-4)$
$\Rightarrow \mathrm{f}(\mathrm{x})=\{-1+\mathrm{a} ; \mathrm{x}<4\}$
$\{\mathrm{a}+\mathrm{b} ; \quad \mathrm{x}=4\}$
$\{1+b ; x>4\}$
LHL $=(-1+\mathrm{a})$
$\therefore \mathrm{LHL}=-1+\mathrm{a}$
RHL $=(1+b)$
$\therefore$ RHL $=1+b$
and $f(4)=a+b$
since $f(x)$ is continuous at $x=4$
$\therefore$ LHL $=$ RHL $=\mathrm{f}(4)$
$\Rightarrow-1+\mathrm{a}=1+\mathrm{b}=\mathrm{a}+\mathrm{b}$
consider $\quad 1+\mathrm{h}=\mathrm{a}+\mathrm{b}$

$$
\mathrm{a}=1
$$

and

$$
\begin{aligned}
-1 & =a=a+b \\
b & =-1
\end{aligned}
$$

$\therefore \mathrm{f}(\mathrm{x})$ is continuous at $\mathrm{x}=4$ for $\mathrm{a}=1 \& \mathrm{~b}=-1$. Ans.
Q2. Find value of ' $k$ ' so that $f(x)$ is continues at $x=0$.
$\mathrm{f}(\mathrm{x})=\left\{\frac{1-\cos (k x)}{x \sin x} ; x \neq 0\right\}$

$$
\{1 / 2 ; x=0\}
$$

Sol. 2

$$
\begin{aligned}
\text { LHL } & =\left[\frac{1-\cos (k x)}{x \sin x}\right] \\
\text { put } \mathrm{x} & =0-\mathrm{h}=-\mathrm{h} \text { and } \mathrm{h} \rightarrow 0 \\
\mathrm{LHL} & =\left[\frac{1-\cos (-k h)}{(-h) \sin (-h)}\right] \\
\text { LHL } & =\left[\frac{1-\cos (k h)}{h \sin x}\right] \quad \ldots \ldots\{\therefore \cos (-\mathrm{x})=\cos \mathrm{x}, \sin (-\mathrm{x})=\sin \mathrm{x}\} \\
& =\left[\frac{2 \sin ^{2}\left(\frac{k h}{2}\right)}{h \sin h}\right]
\end{aligned}
$$

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$$
\begin{aligned}
& =\left[\frac{\frac{2 \sin ^{2}\left(\frac{k h}{2}\right)}{\frac{k^{2} h^{2}}{4}} \times \frac{k^{2} h^{2}}{4}}{h \frac{\sin h}{h} \times h}\right] \\
& =\frac{\left[\frac{\sin ^{2}(k h / 2)}{\frac{k^{2} h^{2}}{4}}\right]}{\left(\frac{\sin h}{h}\right)} \times \frac{2 \mathrm{k}^{2}}{4} \\
& =\frac{1}{1} \times \frac{k^{2}}{2}=\frac{k^{2}}{2}
\end{aligned}
$$

$\mathrm{LHL}=\frac{k^{2}}{2}$
similarly $\mathrm{RHL}=\frac{k^{2}}{2}$
$\mathrm{f}(0)=\frac{1}{2}$
since $\mathrm{f}(\mathrm{x})$ is continuous at $\mathrm{x}=0$
$\therefore \mathrm{LHL}=\mathrm{RHL}=\mathrm{f}(0)$

$$
\begin{aligned}
& \frac{k^{2}}{2}=\frac{k^{2}}{2}=\frac{1}{2} \\
\Rightarrow & \frac{k^{2}}{2}=\frac{1}{2} \\
\Rightarrow & k^{2}=1 \\
& k= \pm 1
\end{aligned}
$$

$\therefore \mathrm{f}(\mathrm{x})$ is continuous at $\mathrm{x}=0$ for $k= \pm 1$ Ans
Q3.

$$
\begin{aligned}
\mathrm{f}(\mathrm{x})= & \left\{\frac{1-\cos (4 \mathrm{x})}{x^{2}} ; x<0\right\} \\
& \left\{\frac{\{\mathrm{a} ; \mathrm{x}=0\}}{\sqrt{16+\sqrt{x}-4}} ; x>0\right\}
\end{aligned}
$$

find value of 'a' so that $f(x)$ is continues at $x=0$.
Sol.3.

$$
\begin{aligned}
\text { LHL } & =\left[\frac{1-\cos (4 \mathrm{x})}{x^{2}}\right] \\
\text { put } \mathrm{x} & =0-\mathrm{h}=-\mathrm{h} \text { and } \mathrm{h} \rightarrow 0 \\
\text { LHL } & =\left[\frac{1-\cos (-4 \mathrm{~h})}{-h^{2}}\right] \\
& =\left[\frac{1-\cos (4 \mathrm{~h})}{h^{2}}\right] \\
& =\left(\frac{2 \sin ^{2}(2 \mathrm{~h})}{h^{2}}\right) \\
& =\left[\frac{2 \sin ^{2}(2 \mathrm{~h})}{4 \mathrm{~h}^{2}} \times 4\right] \\
& =8\left(\frac{\sin ^{2}(2 \mathrm{~h})}{4 \mathrm{~h}^{2}}\right) \\
& =8 \times 1=8 \quad \ldots \ldots .\left(\frac{\sin ^{2} x}{x^{2}}=1\right)
\end{aligned}
$$

$\therefore$ LHL $=8$
$\mathrm{RHL}=\left(\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}\right)$
$\therefore$ RHL $=\left[\frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4}\right]$
rationalize

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$$
\begin{aligned}
& \begin{aligned}
& \mathrm{RHL}=\left[\frac{\sqrt{h}}{\sqrt{16+\sqrt{h}}-4} \times \frac{(\sqrt{16+\sqrt{h}}+4)}{(\sqrt{16+\sqrt{h}}+4)}\right] \\
&=\left[\frac{\sqrt{h}(\sqrt{16+\sqrt{h}}+4)}{16+\sqrt{h}-16}\right] \\
&=[\sqrt{16+\sqrt{h}}+4] \\
&=4+4 \\
& \text { RHL }=8 \\
& \text { Now } \mathrm{f}(0)=\mathrm{a} \\
& \text { since } \mathrm{f}(\mathrm{x}) \text { is continuous at } \mathrm{x}=0 \\
& \text { LHL }=\text { RHL }=\mathrm{f}(0) \\
& \Rightarrow 8=8=\mathrm{a} \quad \ldots \mathrm{a}=8 \\
& \therefore \mathrm{f}(\mathrm{x}) \text { is can't at } \mathrm{x}=0 \text { for } \mathrm{a}=8 \text { Ans. }
\end{aligned}
\end{aligned}
$$

Q4. Determine the value of $\mathrm{a}, \mathrm{b}$ and c so that the function is continues at $\mathrm{x}=0$.

$$
\begin{aligned}
\mathrm{f}(\mathrm{x})= & \left\{\frac{\sin (a+1) x+\sin x}{(x)} ; x<0\right\} \\
& \left\{\frac{\mathrm{c} \quad ; \quad \mathrm{x}=0\}}{b x^{3 / 2}} ; x>0\right\}
\end{aligned}
$$

Sol. 4
LHL $=\left[\frac{\sin (a+1) x+\sin x}{x}\right]$
put $\mathrm{x}=0-\mathrm{h}=-\mathrm{h}$ and $\mathrm{h} \rightarrow 0$
LHL $=\left[\frac{\sin (a+1)(-h)+\sin (-h)}{-h}\right]$
$=\left[\frac{-\sin (a+1) h-\sin h}{-h}\right]$
$=\left[\frac{\sin (a+1)+\sin h}{h}\right]$
$=\left(\frac{\sin (a+1) h}{h}+\frac{\sin h}{h}\right)$
$=\left(\frac{\sin (a+1) h}{h(a+1)} \times(a+1)+\frac{\sin h}{h}\right)$
$=(a+1)\left(\frac{\sin (a+1) h}{h(a+1)}\right)+\left(\frac{\sin h}{h}\right)$
$=(a+1) 1+1 \quad \ldots \ldots\left\{\because\left(\frac{\sin x}{h}\right)=1\right\}$
$\mathrm{LHL}=\mathrm{a}+2$
RHL $=\left(\frac{\sqrt{x+b x^{2}}-\sqrt{x}}{b x^{3 / 2}}\right)$
put $\mathrm{x}=0+\mathrm{h}=\mathrm{h}$ and $\mathrm{h} \rightarrow 0$
$\mathrm{RHL}=\left(\frac{\sqrt{x+b h^{2}}-\sqrt{h}}{b h^{3 / 2}}\right)$
$=\left(\frac{\sqrt{h} \sqrt{1+b h}-\sqrt{h}}{b h \sqrt{h}}\right)$
$=\left(\frac{\sqrt{h} \sqrt{1+b h}-1}{b h \sqrt{h}}\right)$
Rationalize

$$
\begin{aligned}
& =\left(\frac{(\sqrt{1+b h}-1)(\sqrt{1+b h}+1)}{b h(\sqrt{1+b h}+1)}\right) \\
& =\left(\frac{1+b h-1}{b h(\sqrt{1+b h}+1)}\right) \\
& =\left(\frac{1}{\sqrt{1+b h}+1}\right)=\frac{1}{1+1}=\frac{1}{2}
\end{aligned}
$$

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$\mathrm{RHL}=\frac{1}{2}$
$\mathrm{f}(0)=\mathrm{c}$
since $f(x)$ is continuous at $x=0$

```
\(\therefore \quad \mathrm{LHL}=\mathrm{RHL}=\mathrm{f}(0)\)
    \(\Rightarrow a+2=\frac{1}{2}=c\)
    \(\Rightarrow \quad a+2=\frac{1}{2}\) and \(c=\frac{1}{2}\)
\(a=-\frac{3}{2}\) and \(\quad c=\frac{1}{2} \quad\) and \(\quad \mathrm{b}=\mathrm{R}-\{0\}\)
```

$\qquad$ $\{\because$ for $b=0: f(x)$ does not exist $\}$. Ans.

Q5. If the function $f(x)$ is continues at $x=0$. Find the value of $k$.

$$
\begin{aligned}
& \mathrm{f}(\mathrm{x})=\left\{\frac{\log (1+a x)-\log (1-b x)}{(x)} ; x \neq 0\right\} \\
&\{\mathrm{k} ; \mathrm{x}=0\}
\end{aligned}
$$

Sol. 5 RHL $=\left[\frac{\log (1+a x)-\log (1-b x)}{x}\right]$

$$
\text { put } \mathrm{x}=0+\mathrm{h} \text { and } \stackrel{\sim}{\mathrm{h}} \rightarrow 0
$$

$$
\mathrm{RHL}=\left[\frac{\log (1+a h)-\log (1-b h)}{h}\right]
$$

$$
=\left[\frac{\log (1+a h)}{h}-\frac{\log (1-b h)}{h}\right]
$$

$$
=\left[\frac{\log (1+a h)}{a h} \times a-\frac{\log (1+(-b h))}{(-b h)} \times(-b)\right]
$$

$$
=a\left(\frac{\log (1+a h)}{a h}\right)+b\left(\frac{\log (1+(-b h))}{(-b h)}\right)
$$

$$
=a(1)+b(1)
$$

RHL $=a+b$

$$
\left(\frac{\log (1+x)}{x}=1\right)
$$

$\mathrm{f}(0)=\mathrm{k}$
since $f(x)$ is continuous at $x=0$
$\therefore \mathrm{RHL}=\mathrm{f}(0) \quad \Rightarrow \mathrm{a}+\mathrm{b}=\mathrm{k}$
$\therefore \mathrm{k}=\mathrm{a}+\mathrm{b} \quad$ Ans.
Q6. Prove that the greatest integer function $[\mathrm{x}]$ is discontinues at all integral points.
Sol. 6 We have $f(x)=(x)$
let k be only integer i.e. $\mathrm{k} \leftarrow \mathrm{z}$
then $\mathrm{f}(\mathrm{x})=[\mathrm{x}]=\{\mathrm{k}-1$; if $\mathrm{k}-1 \leq \mathrm{x}<\mathrm{k}\}$
$\mathrm{LHL}=(\mathrm{k}-1)$
LHL $=\mathrm{k}-1$
RHL $=(k)$
$\mathrm{RHL}=\mathrm{k}$
and $f(k)=k$
since $\mathrm{LHL}=\mathrm{RHL} \therefore \mathrm{f}(\mathrm{x})$ is discontinuous at ' k ' i.e. all integral points $(\because \mathrm{k} \leftarrow \mathrm{z})$. Ans.
Q7. Show that the function $g(x)=x-[x]$ is discontinues at all integral points.
Sol. 7 We have $g(x)=x-[x]$
let k be any integer i.e $\mathrm{k} \leftarrow \mathrm{z}$
$g(x)=\{x-(k-1) ;$ if $k-1 \leq x<k\}$ $\{x-k ;$ if $k \leq x<k+1\}$
LHL $=(x-(k-1)$
put $x=k-h=-h \& h \rightarrow 0$
$\ldots \mathrm{LHL}=(\mathrm{k}-\mathrm{h}-\mathrm{k}+1)$
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LHL $=-\mathrm{h}+1$
RHL $=(\mathrm{x}-\mathrm{k})$
put $\mathrm{x}=\mathrm{k}+\mathrm{h}$ \& $\mathrm{h} \rightarrow 0$
$\ldots$ RHL $=(\mathrm{k}+\mathrm{h}-\mathrm{k})$
RHL $=0$
$\mathrm{f}(\mathrm{x})=\mathrm{k}-\mathrm{k}=0$
since $\mathrm{LHL} \neq \mathrm{RHL} \ldots \mathrm{f}(\mathrm{x})$ is discontinuous at all integral points ( $\ldots \mathrm{k} \leftarrow \mathrm{z}$ ). Ans.
Q8. Discus the continuity of the function $f(x)=|x-3|-|x-1|$
Sol. 8 We have $f(x)=|x-3|-|x-1|$
first arrange modules so that their critical points are in ascending order.
i.e $\quad f(x)=-|x-1|+|x-3|$
$f(x)=\{+(x-1)-(x-3) ; x<1\}$
$\{-1(x-1)-(x-3) ; 1 \leq x<3\}$
$\{-(x-1)+(x-3) ; x \geq 3\}$
$f(x)=\{+2 ; x<1\}$
$\{-2 x+4 ; 1 \leq x<3\}$ $\{-2 ; x \geq 3\}$
when $\mathrm{x}<1$
$f(x)=2$ which is a constant function, which is everywhere continuous. $\therefore f(x)$ is continuous when $\mathrm{x}<1$.
when $1<x<3$
$f(x)=-2 x+4$ which is a polynomial function, which is everywhere continuous. $f(x)$ is
continuous when
$1<\mathrm{x}<3$.
when $x>3$
$f(x)=-2$ which is a constant function, which is everywhere continuous. $\therefore f(x)$ is continuous when $\mathrm{x} \rightarrow 3$
Now Continuity at $\mathrm{x}=1$
$\mathrm{LHL}=(2) \quad \Rightarrow \mathrm{LHL}=2$
RHL $=(-2 x+4)$
put $\mathrm{x}=1+\mathrm{h} \& \mathrm{~h} \rightarrow 0$
$\therefore$ RHL $=(-2(1+\mathrm{h})+4)$
RHL $=-2+4=2$
$\mathrm{f}(1)=-2(1)+4=2$
LHL $=$ RHL $=\mathrm{f}(1)=2$
$\therefore \mathrm{f}(\mathrm{x})$ is also continuous at $\mathrm{x}=1$
Now continuity at $x=3$
LHL $=(-2 x+4)$
put $\mathrm{x}=3-\mathrm{h}$ and $\mathrm{h} \rightarrow 0$
LHL $=(-2(3-\mathrm{h})+4)$
$\Rightarrow$ LHL $=-6+4=-2$
$\mathrm{RHL}=(-2)$
$f(3)=-2$
$\mathrm{LHL}=\mathrm{RHL}=\mathrm{f}(3)=-2$
$\therefore \mathrm{f}(\mathrm{x})$ is also continuous at $\mathrm{x}=3$
$\therefore \mathrm{f}(\mathrm{x})$ is continuous everywhere (or) there is no point of discontinuity.
(Ans.)

Q9. Show that the function $f(x)|1-x+|x|$ is a continues function .
Sol. 9 Let $g(x)=|-x+|x|$
and $h(x)=|x|$
Now $(\log )(x)=h(g(x))$
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$$
\begin{align*}
= & h(1-x+|x|) \\
(\log )(x) & =|1-x+|x| \\
\Rightarrow(\log )(x) & =f(x) \quad \ldots(1) \tag{1}
\end{align*}
$$

Now $\mathrm{h}(\mathrm{x})=|\mathrm{x}|$ is sum of polynomial and modulus function and sum of two continuous functions is also continuous. $\therefore \mathrm{g}(\mathrm{x})$ is continuous everywhere and composite function of two continuous function is also continuous here $\mathrm{f}(\mathrm{x})$ being a composite function of $\mathrm{h}(\mathrm{x})$ and $\mathrm{g}(\mathrm{x}) \quad \ldots . .\{$ from (i) is also continuous $\}$ Ans.

Q10. Examine that $\sin |x|$ is a continues function.
Sol. 10 Let $f(x)=\sin |x|$
again let $g(x)=|x|$ and $h(x)=\sin x$
Now, $(\operatorname{hog})(\mathrm{x})=h((g(x))$

$$
=h(|x|)
$$

$=\sin |x|$
$\operatorname{hog}(x)=f(x)$
$g(x)=|x|$; which is a modulus function and it is continuous everywhere. $h(x)=\sin x$; is a sine function and it is continuous everywhere.
and composite function of two continuous function is also continuous. here, $f(x)$ being a composite function of $g(x)$ and $h(x)$ is a also continuous.

