CBSE Class 12 Mathematics Differentiation Worksheet

Inverse Trigonometric Diff.

Q1. $y = \cos^{-1}\left(\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right)$. Find $\frac{dy}{dx}$.

Sol.1 We have, $y = \cos^{-1}\left(\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right)$

put $x = \sin\theta$

$$y = \cos^{-1}\left(\frac{2\sin\theta-3\cos\theta}{\sqrt{13}}\right)$$

$$\Rightarrow \quad y = \cos^{-1}\left(\frac{2}{\sqrt{13}}\sin\theta - \frac{3}{\sqrt{13}}.\cos\theta\right)$$

let $\sin\alpha = \frac{2}{\sqrt{13}} \& \cos\beta = \frac{3}{\sqrt{13}}$ ......$\left\{\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{\frac{1-4}{13}} = \frac{3}{\sqrt{13}}\right\}$

$$\Rightarrow \quad y = \cos^{-1}(\sin\alpha.\sin\theta - \cos\alpha.\cos\theta)$$

$$\Rightarrow \quad y = \cos^{-1}(-(\cos\theta.\cos\alpha - \sin\theta.\sin\alpha))$$

$\Rightarrow \quad y = \cos^{-1}(-\cos(\theta+\alpha))$ ......{cos A . cos B – sin A – sin B = cos(A + B)}

$\Rightarrow \quad y = \pi - \cos^{-1}(\cos(\theta+\alpha))$ ....{$\cos^{-1}(-x) = \pi - \cos^{-1} x$}

$$\Rightarrow \quad y = \pi - (\theta+\alpha).$$

$\Rightarrow \quad y = \pi - \sin^{-1}x\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ {Constant}

Diff w.r.t x

$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$ Ans.

Q2. $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$. Find $\frac{dy}{dx}$.

Sol.2 $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

$$\Rightarrow y = \sin^{-1}\left(\frac{2.2^x}{1+(2^x)^2}\right)$$

Put $2^x = \tan\theta$

$$\Rightarrow y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$\Rightarrow y = \sin^{-1}(\sin(2\theta))$$

$\Rightarrow y = 2\theta$

$\Rightarrow y = 2\tan^{-1}(2^x)$

Diff w.r.t. x

$\Rightarrow \frac{dy}{dx} = 2.\frac{1}{1+(2^x)^2}.2^x.\log_2$ ....$\left\{\frac{d}{dx}(a^x) = a^x \log a\right\}$

$\Rightarrow \frac{dy}{dx} = \frac{2^{x+1}.\log_2}{1+4^x}$ Ans

Q3. $y = \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$. Find $\frac{dy}{dx}$.

Sol.3 We have, $y = \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$

put x = sin A and $\sqrt{x}$= sin B

$\Rightarrow \quad y = \sin^{-1}(\sin A\sqrt{1-\sin^2 B} - \sin B\sqrt{1-\sin^2 A})$

$\Rightarrow \quad y = \sin^{-1}(\sin A.\cos B - \sin B.\cos A)$

$\Rightarrow \quad y = \sin^{-1}(\sin(A-B))$

$\Rightarrow \quad y = A - B$

$\Rightarrow \quad y = \sin^{-1}x - \sin^{-1}\sqrt{x}$

Diff w.r.t. x

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}}.\frac{1}{\sqrt{2\sqrt{x}}}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$ Ans.

Diff. Of A Function w.r.t. Another Function

Q4. Diff. $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$.

Sol.4 Let $u = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

put $x = \sin\theta$

$\Rightarrow u = \tan^{-1}\left(\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}\right)$

$\Rightarrow u = \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$

$\Rightarrow u = \tan^{-1}(\cot\theta)$

put $x = \sin\theta$

$\Rightarrow v = \cos^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$\Rightarrow v = \cos^{-1}(2\sin\theta.\cos\theta)$

$\Rightarrow v = \cos^{-1}(\sin(2\theta))$

$\Rightarrow v = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\Rightarrow v = \frac{\pi}{2} - 2\theta$

$\Rightarrow u = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \theta\right)\right)$

$\Rightarrow u = \frac{\pi}{2} - \theta$

$\Rightarrow u = \frac{\pi}{2} - \sin^{-1}x$

Diff w.r.t. x

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

let $v = \cos^{-1}(2x\sqrt{1-x^2})$

$\Rightarrow v = \frac{\pi}{2} - 2\sin^{-1}x$

Diff w.r.t.

$\frac{dv}{dx} = 0 - \frac{2}{\sqrt{1-x^2}} = -\frac{2}{\sqrt{1-x^2}}$

Now $\frac{du}{dx} = \frac{du/dx}{dv/dx} \Rightarrow \frac{du}{dv} = \frac{\frac{-1}{\sqrt{1-x^2}}}{\frac{-2}{\sqrt{1-x^2}}} = \frac{1}{2}$

Q5 Diff $\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$ w.r.t. $\sqrt{1-a^2x^2}$.

Sol.5 Let $u = \sin^{-1}(2ax\sqrt{1-a^2x^2})$

put $ax = \sin\theta$

$u = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$\Rightarrow u = \sin^{-1}(2\sin\theta.\cos\theta)$

$\Rightarrow u = \sin^{-1}(\sin(2\theta))$

$\Rightarrow u = 2\theta$

$\Rightarrow u = 2\sin^{-1}(ax)$

Diff w.r.t. x

$\frac{du}{dx} = \frac{2}{\sqrt{1-a^2x^2}}.(a) = \frac{2a}{\sqrt{1-a^2x^2}}$

let $v = \sqrt{1-a^2x^2}$

Diff w.r.t. x

$\frac{dv}{dx} = \frac{1}{2\sqrt{1-a^2x^2}}(-2a^2x)$

$\frac{dv}{dx} = \frac{-a^2x}{\sqrt{1-a^2x^2}}$

Now $\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\frac{2a}{\sqrt{1-a^2x^2}}}{\frac{-a^2x}{\sqrt{1-a^2x^2}}}$

$\Rightarrow \frac{2a}{-a^2x}$

$\therefore \frac{du}{dv} = \frac{-2}{ax}$ Ans.

Q6. Diff. $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ w.r.t. $\sec^{-1} x$.

Sol.6 Let $u = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$\Rightarrow u = \tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right).\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^{x}\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$$

$$\Rightarrow u = \frac{\pi}{4}-\frac{x}{2}$$

Diff w.r.t. x

$$\frac{du}{dx} = -\frac{1}{2}$$

let $v = sec^{-1}x$

Diff w.r.t. x

$$\frac{dv}{dx} = \frac{1}{x\sqrt{x^2-1}}$$

Now $\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{-1/2}{\frac{1}{x\sqrt{x^2-1}}}$

$\frac{du}{dv} = \frac{-x\sqrt{x^2-1}}{2}$ Ans.

Continuity & Differentiability

Q7. If $f(x)$ is continues at x = 1. Find the values of a and b.

$f(x) = \{3ax + b \; ; \; x > 1\}$

$\{3ax + b \; ; \; x = 1\}$

$\{5ax - 2b \; ; \; x < 1\}$

Sol.7 LHL = $(5ax - 2b)$

Put $x = 1 - h$ and $h \to 0$

LHL = $(5a(1-h) - 2b)$

$\Rightarrow$LHL = $5a - 2b$

RHL = $(3ax + b)$

put $x = 1 + h$ and $h \to 0$

RHL = $[3a(1 + h) + b]$

$\Rightarrow$RHL = $[3a + b]$

Now $f(1) = 11$

Since f(x) is continuous at $x = 1$

$\therefore$ LHL = RHL = f(1)

$\Rightarrow 5a - 2b = 3a + b = 11$

consider $5a - 2b = 11$

and $3a + b = 11$

solving these equations we get

$a = 3$ and $b = 2$

$\therefore$ $f(x)$ is continuous at $x = 1$ for $a = 3$ & $b = 2$

Q8. The function f(x) is continues on [0, 8]. find the value of 'a' and 'b'.

$f(x) = \{x^2 + ax \ ; \ 0 \le x < 2\}$

$\{3x + 2 \ ; \ 2 \le x \le 4\}$

$\{2ax + 5b \ ; 4 < x \le 1\}$

Sol.8 Since f(x) is also continuous in [0 , 8]

$\therefore$ f(x) is also continuous at $x = 2$ and $x = 4$

continuously at $x = 2$

LHL = $(x^2 + ax + b)$

put $x = 2 - h$ and $h \to 0$

$\therefore$ LHL = $[(2 - h)^2 + a(2 - h) + b]$

LHL = $4 + 2a + b$

RHL = $(3x + 2)$

put $x = 2 + h$ & $h \to 0$

$\Rightarrow$RHL = $(3(2 + h) + 2)$

$\Rightarrow$RHL = 8

$f(2) = 3(2) + 2 = 8$

we have , LHL = RHL = f(2)

$\Rightarrow 4 + 2a + b = 8 = 8$

$\Rightarrow 2a + b = 4$ ……(1)

continuity at $x = 4$

LHL = $(3x + 2)$

put x = 4 – h and h → 0

LHL

= (3(4 – h) + 2)

∴ LHL = 14

RHL = (2ax + 5b)

put x = 4 + h and h→ 0

⇒RHL (2a(4 + h) + 5b)

⇒RHL = 8a + 5b

Now f(4) = 3(4) + 2 = 14

we have , LHL = RHL = f(4)

⇒14 = 8a + 5b = 14

⇒8a + 5b = 14 ……(2)

solving (1) & (2)

we get a = 3 & b = -2

∴ f(x) is continuous in [0 , 8] for a = 3 & b = -2 (Ans.)

Q9 If f(x) is continues at x = 2. Find the value of a and b.

f (x) = $\left\{\frac{1-\sin^x}{3\cos^2 x}; x < \frac{\pi}{2}\right\}$

$\left\{a; x = \frac{\pi}{2}\right\}$

$\left\{\frac{b(1-\sin x)}{(\pi-2x)^2}; x > \frac{\pi}{2}\right\}$

Sol.9 LHL = $\left[\frac{1-\sin^3 x}{3\cos^2 x}\right]$

put $x = \frac{\pi}{2} - h$ and $h \to 0$

∴ LHL = $\left[\frac{1-\sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)}\right]$

= $\left[\frac{1-\cos^3 h}{3\sin^2 h}\right]$

= $\left[\frac{(1-\cos h)(1+\cos^2 h+\cos h)}{3(1+\cos^2 h)}\right]$ ……$\{a^3 - b^3 = (a-b)(a^2+b^2+ab)\}$

= $\left[\frac{(1-\cos h)(1+\cos^2 h+\cos h)}{3(1+\cos h)(1-\cos h)}\right]$

= $\left[\frac{1+\cos^2 h\cos h}{3(1+\cos h)}\right]$

LHL $= \frac{1+1+1}{3(1+1)} = \frac{1}{2}$

RHL $= \left[\frac{b(1-\sin x)}{(\pi-2x)^2}\right]$

put $x = \frac{\pi}{2} + h \& h \to 0$

RHL $= \left[\frac{b\left(1-\sin\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)^2\right)}\right]$

$= \left[\frac{b(1-\cos h)}{(\pi-\pi-2h)^2}\right]$

$= \left(\frac{b.2\sin 2(h/\pi)}{4h^2}\right)$

$\Rightarrow$RHL $= \left[\frac{2b.\sin^2(h/2)}{4\frac{h^2}{4}\times 4}\right]$

$= \frac{2b}{16}\left(\frac{\sin^2(h/2)}{h^2/4}\right)$

RHL $= \frac{b}{8}$ .....$\left\{\left(\frac{\sin^2 x}{x^2}\right) = 1\right\}$

$f\left(\frac{\pi}{2}\right) = a$

since f(x) is continuous at x = 2

$\therefore$ LHL = RHL = f(2)

$\Rightarrow \frac{1}{2} = \frac{b}{8} = a$

$\Rightarrow b = 4 \text{and} a = \frac{1}{2}$

$\therefore$ f(x) is continuous at x = 2 if $a = \frac{1}{2}$ and $b = 4$ Ans.

Q10. Find the value of 'a' so that f(x) is continues at $x = 0$.

f (x) = $\left\{a\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right); x \le 0\right\}$

$\left\{\frac{\tan x - \sin x}{x^3}; x > 0\right\}$

Sol.10 LHL = $\left[a\sin\frac{\pi}{2}(x+1)\right]$

put x = 0 – h = -h and h → 0

$\therefore$ LHL =

$= a\sin\left(\frac{\pi}{2}\right)$

LHL = a

RHL $= \left(\frac{\tan x - \sin x}{x^3}\right)$

put $x = 0 + h = h$ and $h \to 0$

$\therefore$ RHL $= \left[\frac{\tan h - \sin h}{h^3}\right]$

$= \left[\frac{\frac{\sin h}{\cos h} - \sin h}{h^3}\right]$

$= \left[\frac{\sin h - \sin h.\cos h}{h^3.\cos h}\right]$

$= \left[\frac{\sin h(1 - \cos h)}{h^3.\cos h}\right]$

$= \left[\frac{\sin h.2\sin^2(h/2)}{h^3.\cos h}\right]$

$= \left[\frac{\sin h}{h} \cdot \frac{2\sin^2(h/2)}{\frac{h^2}{4} \times 4} \cdot \frac{1}{\cos h}\right]$

$= \frac{2}{4}\left(\frac{\sin h}{h}\right) \cdot \left(\frac{\sin^2(h/2)}{h^2/4}\right) \cdot \left(\frac{1}{\cos x}\right)$

$= \frac{1}{2}(1)(1)(1)$ …………$\left\{\left(\frac{\sin x}{x}\right) = 1 \& (\cos x) = 1\right\}$

RHL $= \frac{1}{2}$

f(0) $= a\sin\frac{\pi}{2}(0 + 1) = a\sin\frac{\pi}{2} = a$

since f(x) is continuous at x = 0

LHL = RHL = $f(0)$

$\Rightarrow a = \frac{1}{2} = a$

$\therefore\ a = \frac{1}{2}$ Ans.