CBSE Class 12 Mathematics Differentiation Worksheet

Implicit Functions And General Differentiation

Q1. If $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \log\sqrt{1-x^2}$, show that $\frac{dy}{dx} = \frac{\sin^{-1}x}{(1-x^2)^{3/2}}$.

Sol.1 We have , $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{1}{2}\log(1-x^2)$

Diff w.r.t. x

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2}.\left(x.\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x\right)-x\sin^{-1}x.\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)} + \frac{1}{2}\frac{1}{(1-x^2)}.(-2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+\sqrt{1-x^2}.\sin^{-1}x+\frac{x^2\sin^{-1}x}{\sqrt{1-x^2}}}{(1-x^2)} - \frac{x}{1-x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x\sqrt{1-x^2}+(1-x^2)\sin^{-1}x+x^2\sin^{-1}x}{\sqrt{1-x^2}(1-x^2)} - \frac{x}{1-x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x\sqrt{1-x^2}+\sin^{-1}x-x^2\sin^{-1}x+x^2\sin^{-1}x-x\sqrt{1-x^2}}{\sqrt{1-x^2}(1-x^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^{-1}x}{(1-x^2)^{3/2}}$$ Ans.

Diff. Of Infinite Series

Q2. $y = x^{x^{\cdots\infty}}$. Find $\frac{dy}{dx}$.

Sol.2 We have ,

$\Rightarrow \quad yb = x^y$ ……….{∵}

taking log on both sides

$\Rightarrow \quad \log y = y\log x$

Diff w.r.t. x

$$\frac{1}{y}.\frac{dy}{dx} = y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} - \log x.\frac{dy}{dx} = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1-y\log x}{y}\right) = \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$$ Ans.

Q3. $y = \frac{\sin x}{1+\frac{\sin x}{1+\frac{\cos x}{1+\ldots\infty}}}$. Find $\frac{dy}{dx}$.

Sol.3 $y = \frac{\sin x}{1+\frac{\cos x}{1+y}}$

$\Rightarrow y = \frac{(1+y)\sin x}{(1+y)\cos x}$

$\Rightarrow y + y^2 + y\cos x = \sin x + y\sin x$

then Diff w.r.t. 'x' (Do Yourself)

Ans. $\frac{dy}{dx} = \frac{(1+y)\cos x + y\sin x}{1+2y+\cos x-\sin x}$

Inverse Trigonometric Diff.

Q4. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right); 0 < x < 1$

Sol.4 $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

put $x = \tan\theta$

$\Rightarrow y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \sin^{-1}(\cos(2\theta))$

$\Rightarrow y = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\Rightarrow y = \frac{\pi}{2} - 2\theta$ ....[0 < x < 1] (Conditions)

[0 < $\tan\theta$ < 1]

[θ < θ < $\frac{\pi}{4}$]

$\Rightarrow y = \frac{\pi}{2} - 2\tan^{-1}x$ [0 < 2θ < $\frac{\pi}{2}$]

Diff w.r.t. x [$\Rightarrow 0 < \frac{\pi}{2} - 2\theta < \frac{\pi}{2}$]

$\Rightarrow \frac{dy}{dx} = 0 - \frac{2}{1+x^2} = \frac{-2}{1+x^2}$ Ans.

Q5. $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right); -1 < x < 1$

Sol.5 $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

put $x = \tan\theta$

$\Rightarrow y = \cos^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \cos^{-1}(\sin(2\theta))$

$\Rightarrow y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$ -1 < x < 1 conditions

$\therefore y = \frac{\pi}{2} - 2\theta$ ...... -1 <$\tan\theta$< 1

$\Rightarrow y = \frac{\pi}{2} - 2\tan^{-1}x$ $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$

$\Rightarrow \frac{dy}{dx} = 0 = \frac{2}{1+x^2} = \frac{-2}{1+x^2}$ $-\frac{\pi}{2} < 2\theta < \frac{\pi}{2}$

$\frac{\pi}{2} > -2\theta > -\frac{\pi}{2}$

$\pi > \frac{\pi}{2} - 2\theta > 0$

$\therefore \frac{\pi}{2} - 2\theta \leftarrow (0, \pi)$

Q6. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$. Find $\frac{dy}{dx}$.

Sol.6 $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$

put $x = \tan\theta$

$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$

$\Rightarrow y = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{sec\theta+1}{\tan\theta}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\frac{1}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{1+\cos\theta}{\sin\theta}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{2\cos^2(\theta/2)}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$

$\Rightarrow y = \tan^{-1}\left(\cot\frac{\theta}{2}\right)$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right)$$

$$\Rightarrow y = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\Rightarrow y = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}x$$

Diff w.r.t. x

$\Rightarrow \frac{dy}{dx} = 0 - \frac{1}{2}.\frac{1}{1+x^2} = \frac{-1}{2(1+x^2)}$ Ans.

Q7. $y = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$. Find $\frac{dy}{dx}$.

Sol.7 $y = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$

$$\Rightarrow y = \tan^{-1}\sqrt{\frac{1+\cos\left(\frac{\pi}{2} - x\right)}{1-\cos\left(\frac{\pi}{2} - x\right)}}$$

$$\Rightarrow y = \tan^{-1}\sqrt{\frac{2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}}$$

$$\Rightarrow y = \tan^{-1}\sqrt{\cot^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}$$

$$\Rightarrow y = \tan^{-1}\left(\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right)$$

$$\Rightarrow y = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2}\right)\right)\right]$$

$$\Rightarrow y = \frac{\pi}{2} - \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow y = \frac{x}{2}$$

Diff w.r.t. x

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$ Ans.

Q8. $y=\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$ Show that $\frac{dy}{dx}$ is independent of x.

Sol.8 We have $y=\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$

$\Rightarrow \quad y=\tan^{-1}\left\{\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right\}$ .....$\left\{\tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}x\right\}$

Divide by $\sqrt{1+\sin x}$

$\Rightarrow \quad y=\tan^{-1}\left\{\frac{1-\sqrt{\frac{1-\sin x}{1+\sin x}}}{1+\sqrt{\frac{1-\sin x}{1+\sin x}}}\right\}$

$\Rightarrow \quad y=\tan^{-1}(1)-\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$ ......$\left\{\tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}x-\tan^{-1}y\right\}$

$\Rightarrow \quad y=\frac{\pi}{4}-\tan^{-1}\sqrt{\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}}$

$\Rightarrow \quad y=\frac{\pi}{4}-\tan^{-1}\sqrt{\frac{2\sin^2\left(\frac{\pi}{2}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}}$

$\Rightarrow \quad y=\frac{\pi}{4}-\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$

$\Rightarrow \quad y=\frac{\pi}{4}-\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$\Rightarrow \quad y=\frac{x}{2}$

Diff w.r.t. x

$\frac{dy}{dx}=\frac{1}{2}$ clearly it is independent of x. Ans.

Q9. $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$; x ∊ (1,00) Find $\frac{dy}{dx}$

Sol.9 $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x=\tan\theta$

$\Rightarrow y=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y=\sin^{-1}(\sin(2\theta))$

$\Rightarrow y=\sin^{-1}(\sin(\pi-2\theta))$

$\therefore y=\pi-2\theta$ ....

$\Rightarrow y=\pi-2\tan^{-1}x$

(Conditions)

$1<x<\infty$

$1<\tan\theta<\infty$

$\frac{\pi}{4}<\theta<\frac{\pi}{2}$

$\frac{\pi}{2}<2\theta<\pi$

Diff w.r.t x $\qquad -\frac{\pi}{2} > -2\theta > -x$

$\Rightarrow \frac{dy}{dx} = 0 - 2.\frac{1}{1+x^2} \qquad \frac{\pi}{2} > \pi - 2\theta > 0$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{1+x^2}$ Ans. $\qquad (\pi - 2\theta) \leftarrow (0, \frac{\pi}{2})$

Q10. $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}$; -1` < x < 1. Find $\frac{dy}{dx}$.

Sol.10 we have $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}$

Divide by $\sqrt{1+x^2}$

$$\Rightarrow y = \tan^{-1}\left\{\frac{1+\sqrt{\frac{1-x^2}{1+x^2}}}{1-\sqrt{\frac{1-x^2}{1+x^2}}}\right\}$$

$$\Rightarrow y = \tan^{-1}(1) + \tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right) \quad \ldots\ldots\left\{\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}x + \tan^{-1}y\right\}$$

put $x^2 = \cos(2\theta)$

$$\Rightarrow y = \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{1-\cos(2\theta)}{1+\cos(2\theta)}}$$

$$\Rightarrow y = \frac{\pi}{4} + \tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}$$

$$\Rightarrow y = \frac{\pi}{4} + \tan^{-1}(\tan\theta)$$

$\therefore \; y = \pi + \theta$ …… -1 < x < 1

$\Rightarrow y = \pi + \frac{1}{2}\cos^{-1}(x^2) \qquad 0 < x^2 < 1$

Diff w.r.t. x $\qquad 0 < \cos(2\theta) < 1$

$\Rightarrow \frac{dy}{dx} = 0 - \frac{1}{2}.\frac{1}{\sqrt{1-x^4}}.(2x) \qquad 0 < 2\theta < \theta$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$

(Ans) $\quad 0 < \theta < \frac{\theta}{2}$