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CBSE Class 12 Mathematics Differentiation Worksheet

Implicit Functions And General Differentiation

Q1. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \tan^{-1}a$, show that $\frac{dy}{dx} = \frac{y}{x}$.

Sol.1 We have , $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \tan^{-1}a$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \cos(\tan^{-1}a)$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = k$ .....{where k = cos(tan^{-1}a) = constant} ....(1)

$\Rightarrow x^2 - y^2 = k(x^2+y^2)$

Diff w.r.t. x

$\Rightarrow 2x - 2y\frac{dy}{dx} = k\left(2x + 2y\frac{dy}{dx}\right)$

$\Rightarrow x - y\frac{dy}{dx} = k\left(x + y\frac{dy}{dx}\right)$

$\Rightarrow x - y\frac{dy}{dx} = kx + ky\frac{dy}{dx}$

$\Rightarrow x - kx = ky\frac{dy}{dx} + y\frac{dy}{dx}$

$\Rightarrow x(1-k) = y\frac{dy}{dx}(k+1)$

$\Rightarrow \frac{dy}{dx} = \frac{x(1-k)}{y(k+1)}$

replace k by .....{from eq. (1)}

$\Rightarrow \frac{dy}{dx} = \frac{x\left[1-\frac{x^2-y^2}{x^2+y^2}\right]}{y\left[\frac{x^2-y^2}{x^2+y^2}+1\right]}$

$\Rightarrow \frac{dy}{dx} = \frac{x\left[\frac{x^2+y^2-x^2+y^2}{x^2+y^2}\right]}{y\left[\frac{x^2-y^2+x^2+y^2}{x^2+y^2}\right]}$

$\Rightarrow \frac{x(2y^2)}{y(2x^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$ Proved.

Q2. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3-y^3)$ show that $\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

Sol.2 We have , $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3-y^3)$

put x^3 = sin A and y^3 = sin B

$\Rightarrow \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$

$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right) = a2\cos\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$

$\Rightarrow \frac{A-B}{2} = \cot^{-1}a$

$\Rightarrow A - B = 2\cot^{-1}a$

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replace A by $\sin^{-1}x^3$ & B by $\sin^{-1}y^3$

$$\therefore \sin^{-1}x^3 - \sin^{-1}y^3 = 2\cot^{-1}a$$

Diff w.r.t. x {RHL is constant}

$$\Rightarrow \frac{1}{\sqrt{1-x^6}}.(3x^2) - \frac{1}{\sqrt{1-y^6}}.\left(3y^3\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{x^2}{\sqrt{1-x^6}} = \frac{y^2}{\sqrt{1-x^6}}.\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$$ (Proved)

Q3. If cos y = x cos(a + y) show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

Sol.3 We have , cos y = x cos(a + y) ……(1)

Diff w.r.t. x {product rule on RHL.}

$$-\sin y.\frac{dy}{dx} = -x.\sin(a+y).\frac{dy}{dx} + \cos(a+y).1$$

$$\Rightarrow x\sin(a+y)\frac{dy}{dx} - \sin y\frac{dy}{dx} = \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx}(x\sin(a+y) - \sin y) = \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{x\sin(a+y) - \sin y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{\frac{\cos y}{\cos(a+y)}.\sin(a+y) - \sin y}$$ …..from eq. $x = \frac{\cos y}{\cos(a+y)}$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin(a+y).\cos y - \cos(a+y\sin y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin(a+y-y)}$$ ….{sin A cos B – cos A sin B = sin(A-B)}

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$ (Proved)

Q4. If x sin(a + y) + sin a . cos(a + y) = 0 , show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

Sol.4 We have ,$x\sin(a+y) + \sin a.\cos(a+y) = 0$ …..(1)

Diff w.r.t. x {sin a constant}

$$x.\cos(a+y).\frac{dy}{dx} + \sin(a+y).1 + \sin a[-\sin(a+y)]\frac{dy}{dx} = 0$$

$$\Rightarrow x\cos(a+y)\frac{dy}{dx} + \sin(a+y) - \sin a.\sin(a+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(x\cos(a+y) - \sin a.\sin(a+y)) = -\sin(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(a+y)}{x\cos(a+y) - \sin a.\sin(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(a+y)}{\frac{-\sin a.\cos(a+y)}{\sin(a+y)}.\cos(a+y) - \sin a\sin(a+y)}$$ …….{from eq. (1) value of x}

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin^2(a+y)}{-\sin a.\cos^2(a+y) - \sin a.\sin^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin^2(a+y)}{-\sin a(\cos^2(a+y) + \sin^2(a+y))}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$ …..$\{\cos^2\theta + \sin^2\theta = 1\}$ (Proved)

Q5. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then show that $\frac{dy}{dx} = \frac{1}{x^3y}$.

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Sol.5 We have , $x^2+y^2=t-\frac{1}{t}$ .....(1)

& $x^4+y^4=t^2+\frac{1}{t^2}$ .....(2)

squaring both sides in eq. (1)

$$x^4+y^4+2x^2y^2=t^2+\frac{1}{t^2}-2$$

$\Rightarrow$ $t^2+\frac{1}{t^2}+2x^2y^2=t^2+\frac{1}{t^2}-2$ ....{from eq. (1)}

$\Rightarrow$ $2x^2y^2=-2$

$\Rightarrow$ $x^2y^2=-1$

$\Rightarrow$ $y^2=\frac{-1}{x^2}$

Diff w.r.t. 'x'

$\Rightarrow$ $2y\frac{dy}{dx}=\frac{2}{x^3}$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{x^3y}$ Ans.

Q6. If $y=\log\left(\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\right)$. Find $\frac{dy}{dx}$.

Sol.6 We have ,$y=\log\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$

Diff w.r.t. x

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}.sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\left(\frac{1}{2}\right)$

$\Rightarrow$ $\frac{dy}{dx}=\frac{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}\times\frac{1}{\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}\times\frac{1}{2}$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right).\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{\sin\left(\frac{\pi}{2}+\frac{x}{2}\right)}$ ....$\{2\sin\theta.\cos\theta=\sin(2\theta)\}$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{\cos x}=secx$ Ans.

Q7. If $y=\log_7(\log_7 x)$. Find $\frac{dy}{dx}$.

Sol.7 We have, $y=\log_7(\log_7 x)$

$\Rightarrow$ $y=\frac{\log(\log_7 x)}{\log 7}$ ......$\left\{\text{change of base}\log b=\frac{\log b}{\log a}\right\}$

Diff w.r.t x

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{\log 7}.\frac{1}{\log_7 x}.\frac{d}{dx}(\log_7 x)$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{\log 7}.\frac{1}{\log_7 x}.\frac{d}{dx}\left(\frac{\log x}{\log 7}\right)$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{\log 7}.\frac{1}{\log_7 x}.\frac{1}{\log 7}.\frac{1}{x}$

$\Rightarrow$ $\frac{dy}{dx}=\frac{1}{x(\log 7)^2.\log_7 x}$ Ans.

Q8. If $y=\sqrt{\frac{1-x}{1+x}}$ show that $(1-x^2)\frac{dy}{dx}+y=0$.

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Sol.8 We have, $y = \sqrt{\frac{1-x}{1+x}}$

taking log on both sides

$\Rightarrow \quad \log y = \frac{1}{2}[\log(1-x) - \log(1+x)]$ ....{using log prop.}

Diff w.r.t. x

$\Rightarrow \quad \frac{1}{y}.\frac{dy}{dx} = \frac{1}{2}\left[\frac{-1}{1-x} - \frac{1}{1+x}\right]$

$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{-1-x-1+x}{1-x^2}\right]$

$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left(\frac{-2}{1-x^2}\right)$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx} = -y$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx} + y = 0$ (Proved)

Q9. If $y = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$. Find $\frac{dy}{dx}$.

Sol.9 We have ,$y = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

Diff w.r.t. x

$$\frac{dy}{dx} = \frac{x}{2}.\frac{1}{2\sqrt{a^2-x^2}}.(-2x) + \sqrt{a^2+x^2}.\frac{1}{2} + \frac{a^2}{2}.\frac{1}{\frac{\sqrt{1-x^2}}{a^2}}.\left(\frac{1}{a}\right)$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-x^2}{2\sqrt{a^2-x^2}} + \frac{\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}.\frac{a}{\sqrt{a^2-x^2}}.\frac{1}{a}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-x^2}{2\sqrt{a^2-x^2}} + \frac{\sqrt{a^2-x^2}}{2} + \frac{a^2}{2\sqrt{a^2-x^2}}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-x^2+a^2-x^2+a^2}{2\sqrt{a^2-x^2}}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2a^2-2x^2}{2\sqrt{a^2-x^2}} = \frac{2(a^2-x^2)}{2\sqrt{a^2-x^2}}$

$\Rightarrow \quad \frac{dy}{dx} = \sqrt{a^2-x^2}$ (Ans)

Q10. If $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{dy}{dx} = x + \frac{y}{x}$

Sol.10 We have ,$y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ .....(1)

$\Rightarrow \quad y\sqrt{1-x^2} = x\sin^{-1}x$

Diff w.r.t. (Product rule on both sides)

$$y.\frac{1}{2\sqrt{1-x^2}}(-2x) + \sqrt{1-x^2}.\frac{dy}{dx} = x.\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x.1$$

$\Rightarrow \quad \frac{-xy}{\sqrt{1-x^2}} + \sqrt{1+x^2}.\frac{dy}{dx} = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x.1$

$\Rightarrow \quad \frac{-xy+(1-x^2)dy/dx}{\sqrt{1-x^2}} = \frac{x+\sqrt{1-x^2}.\sin^{-1}x}{\sqrt{1-x^2}}$

$\Rightarrow \quad -xy + (1-x^2)\frac{dy}{dx} = x + \sqrt{1-x^2}.\left(\frac{y\sqrt{1-x^2}}{x}\right)$ ......from(1), $\sin^{-1}x = y\frac{\sqrt{1-x^2}}{x}$

$\Rightarrow \quad -xy + (1-x^2)\frac{dy}{dx} = x + \frac{y(1-x^2)}{x}$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx} = \frac{x^2+y-x^2y+xy}{x}$

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$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2+y-x^2y+x^2y}{x}$

$\Rightarrow \quad \frac{x^2+y}{x}$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx} = x + \frac{y}{x}$ (proved)

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