# CBSE Class 12 Mathematics Differentiation Worksheet

## Parametric functions

Q1. If $x = 2\cos\theta - \cos(2\theta)$ and $y = 2\sin\theta - \sin(2\theta)$. Show that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

Sol.1 We have, $x = 2\cos\theta - \cos(2\theta)$

Diff w.r.t. $\theta$

$$\frac{dx}{d\theta} = -2\sin\theta + 2\sin(2\theta)\text{start}$$

$$\Rightarrow \frac{dx}{d\theta} 2(\sin(2\theta) - \sin\theta)$$

and $y = 2\sin\theta - \sin(2\theta)$

Diff w.r.t $\theta$

$$\frac{dy}{d\theta} 2\cos\theta - 2\cos(2\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = 2(\cos\theta - \cos(2\theta))$$

Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)}$

$$\Rightarrow \frac{dy}{dx} = \frac{-2\sin\left(\frac{3\theta}{2}\right).\sin\left(\frac{-\theta}{2}\right)}{2\cos\left(\frac{3\theta}{2}\right).\sin\left(\frac{\theta}{2}\right)} \quad ...\{\cos A - \cos B \& \sin A - \sin B \text{ farmula}\}$$

$$\frac{dy}{dx} = \frac{\sin(3\theta/2)}{\cos(3\theta/2)} \quad ....\{\sin(-\theta) = -\sin\theta\}$$

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right) \quad \text{Ans.}$$

Q2. $x = \left(t + \frac{1}{t}\right)^a$ and $y = a^{t+\frac{1}{t}}$. Find $\frac{dy}{dx}$.

Sol.2 We have, $x = \left(t + \frac{1}{t}\right)^a$

Diff w. rt. t

$$\frac{dx}{dt} = a\left(t + \frac{1}{t}\right)^{a-1}.\left(1 - \frac{1}{t^2}\right)$$

and $y = a^{t+\frac{1}{t}}$

Diff w. r. t t

$$\frac{dy}{dt} = a^{t+\frac{1}{t}}.\log a.\left(1 - \frac{1}{t^2}\right) \quad .....\left\{\frac{d}{d^x}(a^x) = a^x \log a\right\}$$

Now $$\frac{dy}{dx} = \frac{dy/_{dt}}{dx/_{dt}} = \frac{a^{t+\frac{1}{t}}.\left(1 - \frac{1}{t^2}\right)}{a\left(t + \frac{1}{t}\right)^{a-1}.\left(1 - \frac{1}{t^2}\right)}$$

$$\frac{dy}{dx} = \frac{a^{t+1/t}.\log a}{a.\left(t + \frac{1}{t}\right)^{a-1}} \quad \text{Ans.}$$

Q3. $x = \frac{\sin^3 t}{\sqrt{\cos(2t)}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos(2t)}}$. Find $\frac{dy}{dx}$.

Sol.3 We have , $x = \frac{\sin^3 t}{\sqrt{\cos(2t)}}$

taking log on both sides

$$\log x = 3\log(\sin t) - \frac{1}{2}\log(\cos 2t) \quad \text{......\{using log properties\}}$$

Diff w.r.t 't'

$$\frac{1}{x}.\frac{dx}{dt} = 3.\frac{1}{\sin t}.\cos t - \frac{1}{x}.\frac{1}{\cos(2t)}(-\sin 2t).2$$

$$\Rightarrow \quad \frac{dx}{dt} = x\left[\frac{3}{\tan t} + \tan(2t)\right]$$

and $\quad y = \frac{\cos^3 t}{\sqrt{\cos(2t)}}$

taking log on both sides

$$\log y = 3\log(\cos t) - \frac{1}{2}\log(\cos(2t)) \text{......\{using log properties\}}$$

Diff w.r.t 't'

$$= \frac{\tan^3 t}{\tan^3 t}\left[\frac{-1+3\tan^2 t}{3-\tan^2 t}\right]$$

$$= \frac{1}{\tan t}\left(\frac{-1+3\tan^2 t}{3-\tan^2 t}\right)$$

$$= \frac{-1+3\tan^2 t}{3\tan - \tan^3 t}$$

$$= \frac{-(1-3\tan^2 t)}{3\tan t - \tan^3 t}$$

$$= \frac{-1}{\tan(3t)} \quad \text{.........}\left\{\tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right\}$$

$$\frac{dy}{dx} = -\cot(3t) \quad \text{Ans.}$$

## Higher Order Derivative

Q4. If $y = \sin^{-1}x$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

Sol.4 We have , $y = \sin^{-1}x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2}.\frac{dy}{dx} = 1$$

Diff again w.r.t x (product rule)

$$\sqrt{1-x^2}.\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{1(-2x)}{2\sqrt{1-x^2}} = 0$$

$$\Rightarrow \quad \sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}.\frac{dy}{dx} = 0$$

LCM

$$\Rightarrow \quad \frac{(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0 \quad \text{Hence Proved}$$

Q5. $y = (\cot^{-1}x)^2$ Show that $(x^2+1)^2.y_2 + 2x(x^2+1)y_1 = 2.$

Sol.5 We have , $y = (\cot^{-1}x)^2$

Diff w.rt. x

$$\frac{dy}{dx} = 2(\cot^{-1}x).\left(\frac{-1}{1+x^2}\right) \quad \text{....}\left\{\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}\right\}$$

$\Rightarrow \quad (1+x^2).\frac{dy}{dx} = -2\cot^{-1}x$

Diff again w.r.t. (product rule on LHS)

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = \frac{2}{1+x^2}$$

$\Rightarrow \quad (1+x^2)^2\frac{d^2y}{dx^2_{2x}}(1+x^2)\frac{dy}{dx} = 2$ (Proved)

Q6. If $y = e^{a\cos^{-1}x}$, show that $(1-x)^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.

Sol.6 We have , $y = e^{a\cos^{-1}x}$

taking log on both sides

$$\log y = a\cos^1 x.\log e$$

$\Rightarrow \quad \log y = a\cos^{-1}x$ ......{∵ log e = 1}

Diff w.r.t. x

$$\frac{1}{y}.\frac{dy}{dx} = \frac{-a}{\sqrt{1-x^2}}$$

$\Rightarrow \quad \sqrt{1-x^2}\frac{dy}{dx} = -ay$ .......(1)

Diff again w.r.t x {product rule on LHS}

$\Rightarrow \quad \sqrt{1-x^2}.\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{1}{2\text{sqrt}1-x^2}(-2x) = -a\frac{dy}{dx}$

$\Rightarrow \quad \sqrt{1-x^2}.\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}.\frac{dy}{dx} = -a\frac{dy}{dx}$

LCM $(\sqrt{1-x^2})$

$\Rightarrow \quad \frac{(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}}{\sqrt{1-x^2}} = -a\frac{dy}{dx}$

$\Rightarrow \quad (1-x^2)y_2 - xy_1 = -a\sqrt{1-x^2}.\frac{dy}{dx}$

$\Rightarrow \quad (1-x^2)y_2 = xy_1 = -a(-ay)$ ....{from eq. (i)}

$\Rightarrow \quad (1-x^2)y_2 - xy_1 = a^2y$ (Proved)

Q7. If $y = A\cos(\log x) + B\sin(\log x)$Show that .

Sol.7 We have , $y = A\cos(\log x) + B\sin(\log x)$ .....(1)

Diff w.r.t x

$$\frac{dy}{dx} = -A.\sin(\log x).\frac{1}{x} + B.\cos(\log x).\frac{1}{x}$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-A\sin(\log x) + B\cos(\log x)}{x}$

$\Rightarrow \quad x\frac{dy}{dx} = -A\sin(\log x) + B\cos(\log x)$

Diff w.r.t. x

$$y.\frac{1}{x} + \log x.\frac{dy}{dx} = 1 - dy$$

$$x.\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -A.\cos(\log x).\frac{1}{x} - B\sin(\log x).\frac{1}{x}$$

$\Rightarrow \quad x^2.\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -[A\cos(\log x) + B\sin(\log x)]$

$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$ ....{from eq. (i)}

$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (Proved)

Q8. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$.

Sol.8 W have , $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$\Rightarrow \quad y\sqrt{1-x^2} = \sin^{-1}x$

Diff wrt x (product rule on LHS)

$$y.\frac{1}{2\sqrt{1-x^2}}(-2x) + \sqrt{1-x^2}.\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$\Rightarrow \quad \frac{-xy}{\sqrt{1-x^2}} + \sqrt{1-x^2}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \quad \frac{-xy+(1-x^2)dy/dx}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \quad (1-x^2)\frac{dy}{dx} - xy = 1$

Diff again w.r.t x

$$(1-x^2).\frac{d^2y}{dx^2} + \frac{dy}{dx}(-2x) - \left[x.\frac{dy}{dx} + y.1\right] = 0$$

$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - x\frac{dy}{dx} - y = 0$

$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$ (Proved)

Q9. If $y = [\log(x+\sqrt{x^2+1})]^2$ show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

Sol.9 We have , $y = [\log(x+\sqrt{x^2+1})]^2$

Diff w.r.t. x

$$\frac{dy}{dx} = 2[\log(x+\sqrt{x^2+1})].\frac{1}{x+\sqrt{x^2+1}}.\left(1 + \frac{1}{2\sqrt{x^2+1}}(2x)\right)$$

$\Rightarrow \quad \frac{dy}{dx} = 2.\log(x+\sqrt{x^2+1}).\frac{1}{x+\sqrt{x^2+1}}.\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2\log(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}$

$\Rightarrow \quad \sqrt{1+x^2}\frac{dy}{dx} = 2\log(x+\sqrt{x^2+1})$

Diff again w.r.t. (product rule on LHS)

$$\sqrt{1+x^2}.\frac{d^2y}{dx^2}\frac{dy}{dx}.\frac{1}{2\sqrt{1+x^2}}(2x) = \frac{2}{x+\sqrt{x^2+1}}.\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)$$

$\Rightarrow \quad \sqrt{1+x^2}.\frac{d^2y}{dx^2} + \frac{x}{\sqrt{1+x^2}}.\frac{dy}{dx} = \frac{2}{x+\sqrt{x^2+1}}.\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$

$\Rightarrow \quad \sqrt{1+x^2}.\frac{d^2y}{dx^2} + \frac{x}{\sqrt{1+x^2}}.\frac{dy}{dx} = \frac{2}{\sqrt{x^2+1}}$

LCM in LHS

$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$ (Proved)

Q10. $y = cosec^{-1}x$, show that $(x^2-1)y_2 + (2x^2-1)y_1 = 0$

Sol.10 We have , $y = \text{cosec}^{-1}x$

Diff w.r.t. x

$$\frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$$

$\Rightarrow \quad (x\sqrt{x^2-1}).\frac{dy}{dx} = -1$

Diff again w.r.t. {product rule on LHS}

$\Rightarrow \quad (x\sqrt{x^2-1}).\frac{d^2y}{dx^2} + \frac{dy}{dx}\left\{x.\frac{1(2x)}{2\sqrt{x^2-1}} + \sqrt{x^2-1}\right\} = 0$

$\Rightarrow \quad x\sqrt{x^2-1}.\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}\right) = 0$

$\Rightarrow \quad x\sqrt{x^2-1}.\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{x^2+x^2-1}{\sqrt{x^2-1}}\right)=0$

LCM.

$\Rightarrow \quad x(x^2-1).\frac{dy}{dx^2}+\frac{dy}{dx}(2\mathrm{x}^2-1)=0$ (Proved)