

TOPIC 7

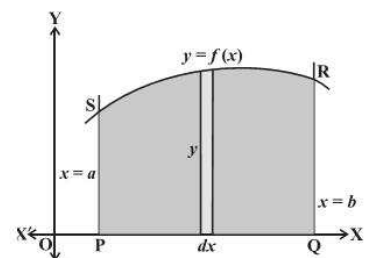
APPLICATIONS OF INTEGRATION

SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	Reference NCERT Text Book Edition 2007
Applications of Integration	(i) Area under <i>Simple Curves</i>	*	Ex.8.1 Q.1,2,5
	(ii) Area of the region enclosed between <i>Parabola and line</i>	***	Ex. 8.1 Q 10,11 Misc.Ex.Q 7
	(iii) Area of the region enclosed between <i>Ellipse and line</i>	***	Example 8, page 369 Misc.Ex. 8
	(iv) Area of the region enclosed between <i>Circle and line</i>	***	Ex. 8.1 Q 6
	(v) Area of the region enclosed between <i>Circle and parabola</i>	***	Ex 8.2 Q1, Misc.Ex.Q 15
	(vi) Area of the region enclosed between <i>Two Circles</i>	***	Example 10, page 370 Ex 8.2 Q2
	(vii) Area of the region enclosed between <i>Two parabolas</i>	***	Example 6, page 368
	(viii) Area of triangle <i>when vertices are given</i>	***	Example 9, page 370 Ex 8.2 Q4
	(ix) Area of triangle <i>when sides are given</i>	***	Ex 8.2 Q5 ,Misc.Ex. Q 14
	(x) <i>Miscellaneous Questions</i>	***	Example 10, page 374 Misc.Ex.Q 4, 12

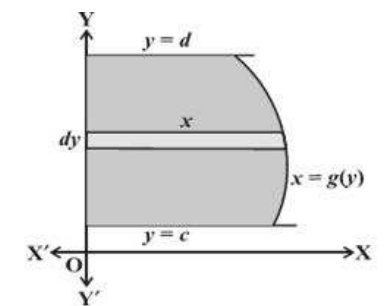
SOME IMPORTANT RESULTS/CONCEPTS

** Area of the region PQRSP = $\int_a^b dA = \int_a^b y \, dx = \int_a^b f(x) \, dx$.



** The area A of the region bounded by the curve $x = g(y)$, y-axis and

the lines $y = c$, $y = d$ is given by $A = \int_c^d x \, dy = \int_c^d g(y) \, dy$



ASSIGNMENTS

(i) Area under *Simple Curves*

LEVEL I

1. Sketch the region of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and find its area, using integration,
2. Sketch the region $\{(x, y) : 4x^2 + 9y^2 = 36\}$ and find its area, using integration.

(ii) Area of the region enclosed between *Parabola and line*

LEVEL II

1. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.
2. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

LEVEL III

1. Find the area of the region : $\{(x, y) : y \leq x^2 + 1, y \leq x + 1, 0 \leq x \leq 2\}$

(iii) Area of the region enclosed between *Ellipse and line*

LEVEL II

1. Find the area of smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and the straight line $\frac{x}{4} + \frac{y}{5} = 1$.

(iv) Area of the region enclosed between *Circle and line*

LEVEL II

1. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

LEVEL III

1. Find the area of the region : $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$

(v) Area of the region enclosed between *Circle and parabola*

LEVEL III

1. Draw the rough sketch of the region $\{(x, y) : x^2 \leq 6y, x^2 + y^2 \leq 16\}$ and find the area enclosed by the region using the method of integration.
2. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

(vi) Area of the region enclosed between *Two Circles*

LEVEL III

1. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

(vii) Area of the region enclosed between *Two parabolas*

LEVEL II

1. Draw the rough sketch and find the area of the region bounded by two parabolas

$4y^2 = 9x$ and $3x^2 = 16y$ by using method of integration.

(viii) Area of triangle *when vertices are given*

LEVEL III

1. Using integration compute the area of the region bounded by the triangle whose vertices are (2, 1), (3, 4), and (5, 2).
2. Using integration compute the area of the region bounded by the triangle whose vertices are (-1, 1), (0, 5), and (3, 2).

(ix) Area of triangle *when sides are given*

LEVEL III

1. Using integration find the area of the region bounded by the triangle whose sides are $y = 2x + 1$, $y = 3x + 1$, $x = 4$.
2. Using integration compute the area of the region bounded by the lines $x + 2y = 2$, $y - x = 1$, and $2x + y = 7$.

(x) Miscellaneous Questions

LEVEL III

1. Find the area of the region bounded by the curves $y = |x - 1|$ and $y = -|x - 1| + 1$.
2. Find the area bounded by the curve $y = x$ and $y = x^3$.
3. Draw a rough sketch of the curve $y = \sin x$ and $y = \cos x$ as x varies from $x = 0$ to $x = \frac{\pi}{2}$ and find the area of the region enclosed by them and x -axis
4. Sketch the graph of $y = |x + 1|$. Evaluate $\int_{-3}^1 |x + 1| dx$. What does this value represent on the graph.
5. Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.
6. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x -axis and between $x = -6$ to $x = 0$.

[CBSE 2011]

Questions for self evaluation

1. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.
2. Find the area bounded by the parabola $y = x^2$ and $y = |x|$.
3. Find the area of the region : $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$
4. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.
5. Find the area of the region : $\{(x, y): x^2 + y^2 \leq 1, x + y \geq 0\}$
6. Find the area lying above the x -axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.
7. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

8. Using integration compute the area of the region bounded by the triangle whose vertices are (2, 1), (3, 4), and (5, 2).

9. Using integration compute the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$, and $x - 3y + 5 = 0$.

10. Sketch the graph of : $f(x) = \begin{cases} |x - 2| + 2, & x \leq 2 \\ x^2 - 2, & x > 2 \end{cases}$.

Evaluate $\int_0^4 f(x) dx$. What does the value of this integral represent on the graph ?