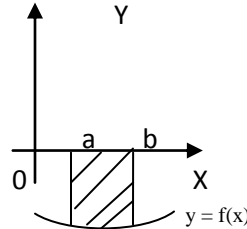
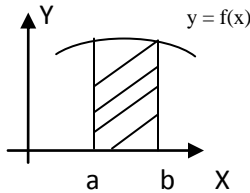


## APPLICATION OF INTEGRALS

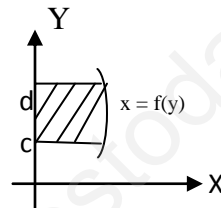
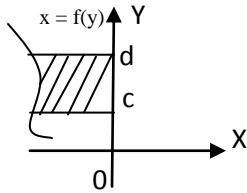
### KEY POINTS TO REMEMBER

- Area bounded by the curve  $y = f(x)$ , the x axis and between the ordinates,  $x = a$  and  $x = b$  is given by

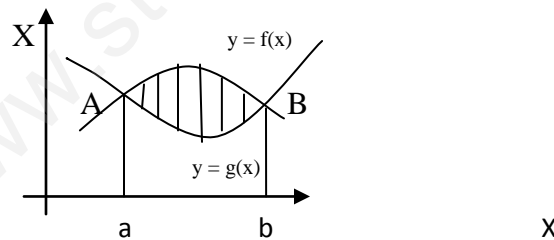
$$\text{Area} = \int_a^b f(x) dx$$



- Area bounded by the curve  $x = f(y)$ , the y axis and between abscissas,  $y = c$  and  $y = d$  is given by  $\int_c^d f(y) dy$

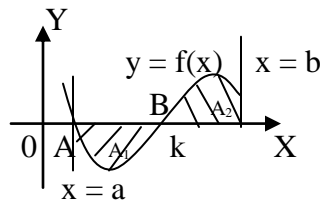


- Area bounded by two curves  $y = f(x)$  &  $y = g(x)$  s. t.  $0 \leq g(x) \leq f(x)$  for all  $x \in [a, b]$  & between the ordinate at  $x = a$  and  $x = b$  is given by  $\int_a^b f(x) - g(x) dx$



$$\text{Required Area} = \int_a^b f(x) - g(x) dx$$

- Area bounded by the following curve given below given by



$$\text{Required Area} = \left| \int_a^k f(x) dx \right| + \int_k^b f(x) dx$$

**ASSIGNMENT**

1. Find the area of the following regions:

i.  $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$

ii.  $\{(x, y) : x^2 + y^2 \leq 2ax, y^2 \geq ax, x \geq 0, y \geq 0\}$

iii.  $\{(x, y) : \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2}\}$

iv.  $\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$

v.  $\{(x, y) : x^2 + y^2 \leq 4, x + y \geq 2\}$

vi.  $\{(x, y) : 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$

vii.  $\{(x, y) : x^2 \leq y \leq |x|\}$

2. Sketch the graph of  $y = |x + 1|$  and evaluate  $\int_{-3}^1 |x + 1| dx$ . What does

$\int_{-3}^1 |x + 1| dx$  represent on the graph?

3. Using the method of integration, find the area of the region bounded by the lines  $3x - 2y + 1 = 0$ ,  $2x + 3y - 21 = 0$  and  $x - 5y + 9 = 0$ .

4. Sketch the region common to the circle  $x^2 + y^2 = 16$  and parabola  $x^2 = 6y$ . Also find the area of the region using integration.

5. Using integration, find the area of the region bounded by the curves  $y = x^2 + 2$ ,  $y = x$ ,  $x = 0$  and  $x = 3$ .

6. Compute the area bounded by the lines  $x + 2y = 2$ ,  $y - x = 1$  and  $2x + y = 7$ .

**ANSWER KEY**

1. i.  $(\frac{\pi}{4} - \frac{1}{2})$  sq. units    ii.  $(\frac{\pi}{4} - \frac{2}{3})a^2$  sq. units    iii.  $\frac{3}{2}(\pi - 2)$  sq. units    iv.  $(\frac{5\pi}{4} - \frac{1}{2})$  sq. units

v.  $(\pi - 2)$  sq. units    vi.  $50/3$  sq. units    vii.  $1/3$  sq. units    2. 4    3.  $13/2$  sq. units

4.  $(\frac{4\sqrt{3}}{3} + \frac{16\pi}{3})$  sq. units    5.  $21/2$  sq. units    6. 6 sq. units