Chapter: - Application of Integration, Differential equations

1 marks question

Q1. Find area under the curve y= $\sqrt{a^2 - x^2}$ included between the lines x=0 and x=a **Ans.** $\frac{\pi a^2}{4}$ sq.units.

Q2.Find area of the region bounded by the curve y^2=4x, y-axis and the line y=3 **Ans.** $\frac{9}{4}$ sq.units.

Q3.Find area of the region bounded by the curve x^3=y, x-axis , line x=-2and x=1 **Ans.** $\frac{4}{3}$ sq.units.

Q4.Find area of the region bounded by the curve y=cosx , x=0and x=π **Ans.** 2 sq.units.

Q5. Find the order and degree of the differential equation:- $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^2y}{d^2x}\right)^2 = 0$. **Ans**. 2, 2

Q6. Find the number of arbitrary constants in the general solution of a differential equation of fourth order. **Ans**. 4

4/6 marks question

Q7. Draw a rough sketch of the region $\{(x, y) : \ y^2 \le 6ax, \ x^2 + y^2 \le 16a^2\}$. Also, find the area of the region sketched, using integration. **Ans.** $4/3(\sqrt{3} + 4\pi)a^2$ sq.units.

Q8. Draw a rough sketch of the curve y = $\sin x$ and y = $\cos x$ as x varies from 0 to π/2 and find the area of the region enclosed by them and x-axis. **Ans**. $(2 - \sqrt{2})$ sq.units..

Q9. Find the area of the region lying between the parabola $y^2 = 4ax$ and $x^2 = 4ay$, where a> 0 . **Ans.** $(16a^2/3)\text{sq. units.}$

Q10.Find the area of the region bounded by the curve y = $\sqrt{1 - x^2}$, line y = x and the positive x-axis. **Ans.** π/8. sq.units.

Q11.Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$. **Ans.** 64/3. sq.units.

Q12. Find the area of the following region: $\{(x, y) : \ x^2 + y^2 \le 2ax, \ y^2 \ge ax, x \ge 0\ , y \ge 0\}$.
Ans. $a^2/12(3\pi - 8)$. sq.units.

Q13.Find the area bounded by the curve $y^2 = 4a^2(x - 3)$ and the line $x = 3, y = 4a$. **Ans.**16a/3.

Q14. Calculate the area of the region enclosed between circles $x^2 + y^2 = 1$ and $(x - 1/2)^2 + y^2 = 1$. **Ans.** $\left[-\frac{2\sqrt{3}+\sqrt{15}}{16} - 2\sin^{-1}\frac{1}{4} + \pi\right]$. sq.units.

Q15.Sketch the graph of $f(x) = \begin{cases} |x-2| + 2 & x \le 2 \\ x^2 - 2 & x > 2 \end{cases}$.Evaluate $\int_0^4 f(x)dx$ .What does the value of this integral represent on the graph? **Ans.** $\frac{62}{3}$ sq.units.

P.T.O.

Q16. Make a rough sketch of the region given below and find its area using integration.
$\{(x, y) : 0 \le y \le x^2 + 3 ; 0 \le y \le 2x + 3, 0 \le x \le 3\}$. **Ans.** 50/3. sq.units.

Q17. Determine the area enclosed between the curve $y = 4x - x^2$ and x-axis. Ans. 32/3.sq.units

Q18.Solve the following differential equations :

(i). $\frac{dy}{dx} = \frac{e^x}{y} \frac{(\sin^2 x + \sin 2x)}{(2 \log y + 1)}$. **Ans.** $y^2 \log y = \sin^2 x . e^x + c.$

(ii). $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0.$ **Ans**. $\sqrt{1 + x^2} + \frac{1}{2}\log\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1} + \sqrt{1 + y^2} = c.$

(iii). $e^x \sqrt{1 - y^2}\, dx + \frac{y}{x}\, dy = 0.$ Ans. $(x - 1)e^x - \sqrt{1 - y^2} = c.$

(iv). $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right) dy = 0.$ **Ans.** $x + ye^{\frac{x}{y}} = c.$

(v). $\frac{dy}{dx} = \cos^3 x \sin^4 x + x\sqrt{2x - 1}\,.$ **Ans.** $y = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + e^x(x - 1) + c.$

(vi). $\frac{dy}{dx} + \frac{4x}{x^2+1} y + \frac{1}{(x^2+1)^2} = 0\,.$ **Ans**. $(x^2 + 1)^2 y = -x + c.$

(vii). $\frac{dy}{dx} + y \cot x = 2x + x^2 \cos x$, given that $y(0) = 0$. **Ans.** $y = x^2.$

(viii). $(x - \sin y)dy + \tan y\, dx = 0, y(0) = 0.$ **Ans.** $y = \sin^{-1} 2x$.

(ix). $ye^y\, dx = (y^3 + 2xe^y)\, dy\, , y(0) = 1$. **Ans.** $x = y^2(e^{-1} - e^{-y})$

(x). $(1 + \sin^2 x)dy + (1 + y^2) \cos x\, dx = 0,\ y(1) = 0$. **Ans.** $\tan^{-1}(\sin x) + \tan^{-1} y = \pi/4.$

(xi). $y - x\frac{dy}{dx} = a\left(y^2 + x^2 \frac{dy}{dx}\right)$, where $x = a\, , y = a.$

Ans. $x(1 - ay)(1 + a^2) = y(ax + 1)(a^2 - 1)$

(xii). $\frac{dy}{dx} = e^{y+x} + e^y x^2.$ **Ans.** $e^x + e^{-y} + \frac{x^3}{3} = c.$

(xiii). $2ye^{x/y}\, dx + \left(y - 2xe^{x/y}\right)dy = 0.$ **Ans.** $2e^{\frac{x}{y}} + \log y = c$.

Q19. Show that $y = ae^{2x} + be^{-x}$ is a solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0.$

Q20. Form the differential equation corresponding to $y^2 = a(b - x^2)$ where a and b are arbitrary constants. **Ans.** $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$.

Q21. Form the differential equation of family of curves given by $xy = Ae^x + Be^{-x} + x^2.$

Ans. $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xy - x^2 + 2.$

Q22. Form the differential equation corresponding to $y^2 = a(b - x)(b + x)$ by eliminating a and b. **Ans.** $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0.$

Q23. Show that the differential equation of which $y = 2(x^2 - 1) + ce^{-x^2}$ is a solution is
$\frac{dy}{dx} + 2xy = 4x^3.$

Q24. Form the differential equation representing the family of curves $y^2 - 2ay + x^2 = a^2$, where a is an arbitrary constants. **Ans.** $(x^2 - 2y^2)\left(\frac{dy}{dx}\right)^2 - 4xy\frac{dy}{dx} - x^2 = 0.$

Q25. Form the differential equation of which $y = \tan^{-1} x + ce^{-\tan^{-1} x}$ is a solution, c being an arbitrary constant. **Ans.** $y + (1 + x^2)\frac{dy}{dx} = 1 + \tan^{-1} x.$

-------------------------------- **Best of Luck** --------------------------------