Assignment on Application of Derivatives

1. A man, 2 meter tall, walks at the rate of $1\frac{2}{3}$ m/s towards a street light which is $5\frac{1}{3}$ meter above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow changing when he is $3\frac{1}{3}$ meter from the base of street light?
2. Two men A and B start with velocities v at the same time from the junctions of two roads inclined at 45^0 to each other. If they travel by different roads, find the rate at which they are being separated.
3. A ladder 5 meter long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 meter/second. How fast is its height on the wall decreasing when the foot of the ladder is 4 meter away from the wall.
4. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/second , then how fast is the slope of curve changing when $x = 3$ units?
5. a) At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to y – axis?

   b) Find the coordinates of the point on the curve $\sqrt{x} + \sqrt{y} = 4$ at which the tangent is equally inclined to axes.
6. Find the condition that the curves $2x = y^2$ and $2xy = k$ intersect orthogonally.
7. Using differentials, find the approximate value of $\sqrt{0.082}$ and $(1.999)^5$.
8. a) Show that $f(x) = 2x + cot^{-1}x + log(\sqrt{1+x^2} - x)$ is increasing in R.

   b) Show that for $a \geq 1$, $f(x) = \sqrt{3}\, sin\, x - 2\, ax + b$ is decreasing in R.
9. Determine the intervals in which the following functions are strictly increasing or strictly decreasing : a) $x^4 - \frac{4x^3}{3}$, b) $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$
10. Show that the function $f(x) = \tan^{-1}(\sin x + \cos x)$ is strictly increasing in $(0, \frac{\pi}{4})$.
11. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
12. A telephone company in a town has 500 subscriber on its list and collects fixed charges of Rs.300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Re. 1 , one subscriber will discontinue the service. Find what increase will bring maximum profit?
13. A sheet of paper for a poster is 1.5 m^2. In area. The margins at the top and bottom are to be 6 cm wide and at the sides, 4 cm wide. What should be the dimensions of the sheet to maximize the printed area.

14. A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs Rs. 5 per cm^2 and the material for the sides costs Rs. 2.50 per cm^2. Find the least cost of the box.
15. If the sum of the surface areas of a cube and a sphere is constant, what is the ratio of an edge of the cube to the diameter of the sphere when the sum of their volumes is minimum?
16. A given quantity of metal is to be cast into a half circular cylinder (i.e. with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : \pi + 2$.
17. The section of a window is a rectangle surmounted by an equilateral triangle. Given that the perimeter is 16 m, find the width of window in order that maximum light may be admitted.
18. Two sides of a triangle are given , find the angle between them such that its area is maximum.
19. The total cost of manufacturing x pocket radios per day is Rs. $\left(\frac{x^2}{4}+35x+25\right)$ and rate at which they may be sold to a distributer is Rs. $\frac{1}{2}(100-x)$ each. What should be the daily output to attain maximum total profit?
20. An isosceles triangle of vertical angle 2Θ is inscribed in a circle of radius r. Show that the area of triangle is maximum when $\Theta = \frac{\pi}{6}$.
21. If a function is defined by $f(x)=\begin{cases} x^2+3x+a, x\le 1 \\ bx+2, x>1 \end{cases}$ is differentiable at each $x \in$ R, then find the values of a and b.
22. For what choices of a and b is the function $f(x)=\begin{cases} bx^2, x\le 1 \\ ax-6, x>1 \end{cases}$ is differentiable at $x=1$.
23. Discuss the differentiability of $f(x)=|x-1|+|x-2|$.
24. Discuss differentiability of $f(x)=x|x|$ at $x=0$.
25. Examine the differentiability of f at $x=2$, where f is defined by

$$f(x)=\begin{cases} x[x], 0\le x<2 \\ (x-1)x, 2\le x<3 \end{cases}$$