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APPLICATION OF INTEGRAL

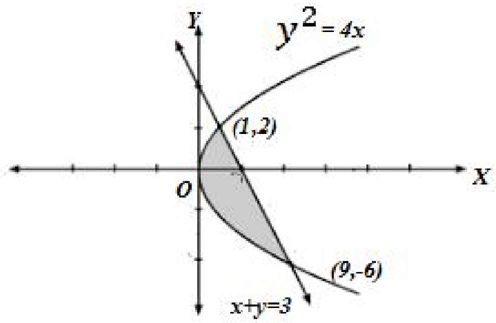
Q.1) Find the area bounded by the curves $y^2 = 4x$ and $x + y = 3$

Sol.1) 1) $y^2 = 4x$

(.) parabola

(.) vertex (0,0)

(.) open $+ve$ x-axis



2) $x + y = 2$

(.) line , point $(0, 3)$ & $(3, 0)$

Intersection Point

Put $y^2 = 4x$ in $x + y = 3$

or $x = \frac{y^2}{4}$

$\Rightarrow \frac{y^2}{4} + y = 3$

$\Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow (y + 6)(y - 2) = 0$

$y = -6$ and $y = 2$

$x = 9$ and $x = 1$

points $(1, 2)$ & $(9, -6)$

New concept = $\int_{y_1}^{y_2} (\text{right curve ka } x) - (\text{left curve ka } x)$

Required area = $\int_{-6}^{2} (3 - y) - \frac{y^2}{4} dy$

$= \frac{1}{4} \int_{-6}^{2} (12 - 4y - y^2)\, dy$

$= \frac{1}{4} \left[12y - 2y^2 - \frac{y^3}{3}\right]_{-6}^{2}$

$= \frac{1}{4} \left[\left(24 - 8 - \frac{8}{3}\right) - \left(-72 - 72 + \frac{216}{3}\right)\right]$

$= \frac{1}{4} \left[16 - \frac{8}{3} + 144 - \frac{216}{3}\right]$

$= \frac{1}{4} \left[160 - \frac{224}{3}\right]$

$= \frac{1}{4} \left[\frac{480 - 224}{3}\right]$

$= \frac{1}{4} \left(\frac{256}{3}\right)$

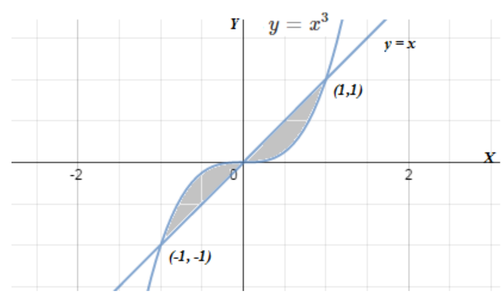
$= \frac{64}{3}$

$\therefore$ Required area = $\frac{64}{3}$ sq. units ans.

Q.2) Find the area bounded by $y = x$ and $y = x^3$.

Sol.2) 1) $y = x$ line passing through (0,0)

2) $y = x^3$.



Points (0,0) , (1,1) , (2,8) , $(-1, -1)$, $(-2, -8)$

Intersection point

Put $y = x$ in $y = x^3$.

$\Rightarrow x^3 = x$

$\Rightarrow x^3 - x = 0$

$\Rightarrow x(x^2 - 1) = 0$

$x = 0, x = 1, x = -1$

$\therefore$ (0,0) , (1,1) & $(-1, -1)$

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$y = 0, y = 1, y = -1$

Required area = $2\int_0^1 (x - x^3)\,dx$

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$

$= 2\left[\left(\frac{1}{2} - \frac{1}{4}\right) - (0 - 0)\right]$

$= 2\left[\frac{2-1}{4}\right] = 2\left(\frac{1}{4}\right) = \frac{1}{2}$

Required area = $\frac{1}{2}$ sq. units ans.

Q.3) Complete the area bounded by the curves $x = a, x = 2, y = 2^x$ and $y = 2x - x^2$

Sol.3) 1) $y = 2x - x^2$ (shifting parabola)

$x^2 - 2x = -y$

$(x-1)^2 - 1 = -y$

$(x-1)^2 = -y + 1$

$(x-1)^2 = -(y-1)$

Vertex (1,1)

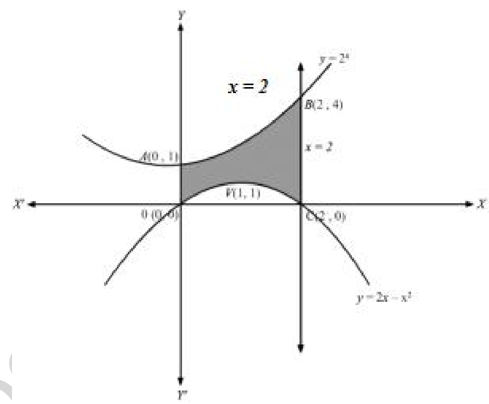
Open $-ve$ y-axis

$y = 2^x$ (exponential curve)

$(0,1), (1,2), (2,4)$ …..

$x = 0$ (equation of y-axis)

$x = 2$ (line parallel to y-axis at (2,0))



Intersection of parabola $y = 2x - x^2$ with x-axis

Put $y = 0$

$\Rightarrow 0 = 2x - x^2$

$\Rightarrow 0 = x(2 - x)$

$x = 0$ and $x = 2$

Required area = $\int_0^2 2^x - (2x - x^2)\,dx$

$= \left(\frac{2^x}{\log 2} - x^2 + \frac{x^2}{3}\right)_0^2$

$= \left(\frac{4}{\log 2} - 4 + \frac{8}{3}\right) - \left(\frac{1}{\log 2} - 0 + 0\right)$

Required area = $\left(\frac{3}{\log 2} - \frac{4}{3}\right)$ sq. units ans.

Q.4) Find the area bounded by curves $y = -1, y = 2, x = y^3$ and $x = 0$

Sol.4)

1) $y = -1$ (line parallel to x-axis at (0, –1))

2) $y = 2$ (line parallel to x-axis (0,2))

3) $x = y^3$ (points $(0,0), (1,1), (8,2), (-1,-1), (-8,-2)$)

4) $x = 0$ (eq. of y-axis)

Picture graph

Required = $\int_{-1}^{0} \left(x^{\frac{1}{3}} - (-1)\right) dx + \int_0^8 \left(2 - x^{\frac{1}{3}}\right) dx$

$= \left[\frac{3}{4}x^{\frac{4}{3}} + x\right]_{-1}^{0} + \left[2x - \frac{3}{4}x^{\frac{4}{3}}\right]_0^8$

$= (0 + 0) - \left(\frac{3}{4}(-1)^{\frac{4}{3}} - 1\right) + \left(16 - \frac{3}{4}(8)^{\frac{4}{3}}\right) - \left[2(0) - \frac{3}{4}(0)^{\frac{4}{3}}\right]$

$= -\left(\frac{3}{4} - 1\right) + \left(16 - \frac{3}{4} \times 16\right) - 0$

$= -\left(-\frac{1}{4}\right) + \left(\frac{64-48}{4}\right)$

$= \frac{1}{4} + \frac{16}{4} = \frac{17}{4}$

∴ Required area = $\frac{17}{4}$ sq. units ans.

Q.5) Find the area bounded by the y-axis, $y = \cos x$, $y = \sin x$ when $0 \le x \le \frac{\pi}{2}$

Sol.5)

1) $y = \cos x$

$x = 0, y = 1;\ x = \frac{\pi}{6}, y = \frac{\sqrt{3}}{2};\ x = \frac{\pi}{6}, y = \frac{1}{2}, x = \frac{\pi}{2}, y = 0$

2) $y = \sin x$

$x = 0, y = 0, x = \frac{\pi}{6}, y = \frac{1}{2}, x = \frac{\pi}{3}, y = \frac{\sqrt{3}}{2}, x = \frac{\pi}{2}, y = 6$

3) y-axis

Intersection point

$y = \sin x$ & $y = \cos x$

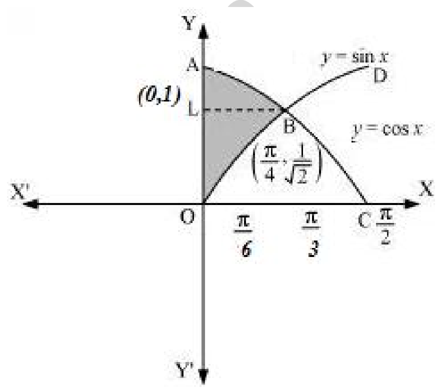
$\sin x = \cos x$

$\Rightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}$

∴ point $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

3) $y = \frac{1}{\sqrt{2}}$



Required area = $\int_0^{\frac{\pi}{4}} (\cos x - \sin x)\, dx$

$= [\sin x + \cos x]_0^{\frac{9}{4}}$

$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] - [0 + 1]$

$= \frac{2}{\sqrt{2}} - 1 = \frac{2-\sqrt{2}}{2}$ sq. units ans.

Q.6) Find the area bounded by the curve $y = \cos x$, $x = 0$ and $x = 2\pi$

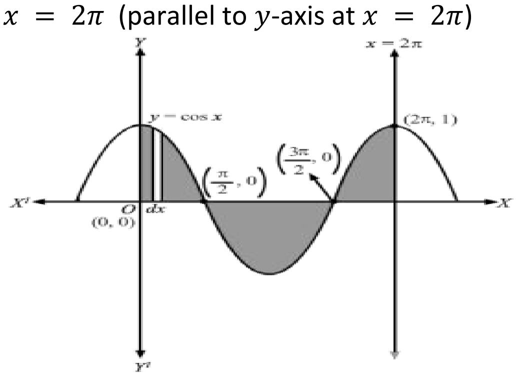
Sol.6)

1) $y = \cos x$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	1	0	-1	0	1

2) $x = 0$ (eq. of y-axis)

3) $x = 2\pi$ (parallel to y-axis at $x = 2\pi$)



Required area = $4\int_0^{\frac{\pi}{2}} (\cos x - 0)\, dx$ ....{due to symmetry of cos x}

	$= 4\,(\sin x)_0^{\frac{\pi}{2}}$ $= 4(1-0)$ Required area $=\ 4\,sq.units$ ans..
Q.7)	Draw a rough sketch of the curve $y\ = 2\cos^2 x$ in $[0,\pi]$ and find the area enclosed by the curve the line $x\ =\ 0\ ,\ x\ =\ \pi$ and x-axis
Sol.7) 1) 2) 3) 4)	$y\ = 2\cos^2 x$ in $[0,\pi]$ $x = 0, y = 2, x = \dfrac{\pi}{2}\ , y = 0\ , x = \pi\ , y =\ 2$ $x\ =\ 0$ (y-axis) $x\ =\ \pi$ x-axis picture Required area = $2\int_0^{\frac{\pi}{2}}(2\cos^2 x - 0)\,dx$ $= 2\int_0^{\frac{\pi}{2}}\frac{2(1+cos\,2x)}{2} - 0\,dx$ $= 2\left[x + \frac{\sin(2x)}{2}\right]_0^{\frac{\pi}{2}}$ $= 2\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - (0+0)\right]$ $= 2\left[\frac{\pi}{2} + 0\right]$ Required area $=\ \pi\,sq.units$ ans.