

**CLASS – XII**

**SUBJECT – MATHEMATICS**

**ASSIGNMENT NO. 4**

1. A particle moves along the curve  $y = x^2 + 2x$  at what points on the curve are the  $x$  &  $y$  co-ordinates of the particle changing at the same rate?
2. A balloon in form of a circular cone surmounted by a hemisphere having a diam. Equal to the height of the cone is being inflated. How fast is its vol. Is changing with respect to the total length  $h$ , when  $h = 9$  cm.
3. The radius of cylinder is increasing at the rate 2cm/sec & its altitude decreasing at the rate of 3cm/sec. Find the rate of change volume where radius is 3 cm & alt. 5 cm.
4. A particle moves along the curve  $y = \frac{2}{3}x^3 + 1$ . Find the points on the curve at which the  $y$ -co-ordinate is changing twice as fast as  $x$ -co-ordinate.
5. Use differentiation to find the approx. values of (i)  $(0.009)^{1/3}$  (ii)  $(0.007)^{1/3}$  (iii)  $(255)^{1/4}$
6. Find approx. value of  $f(5.00)$  if  $f(x) = x^3 - 7x^2 + 15$
7. Find the percentage error in finding the surface area of cubical box if an error 1% is made in measuring the length of edges of the cube.
8. Verify Rolle's Theorem for the functions  $f(x) = (x - 9)^m (x - b)^n$  on the interval  $(a, b)$  where  $m, n$  are +ve integers.
9. Verify Rolle's theorem for (i)  $f(x) = e^x / \sin x - \cos x$  on  $\frac{n}{4}, \frac{5n}{4}$
10. Verify mean value theorem for (i)  $f(x) = 2\sin x + \sin 2x$  on  $(0, \pi)$  (ii)  $f(x) = x^3 - 2x^2 - x + 3$  on  $(0, 1)$  (iii)  $f(x) = 10e^x$  on  $[1, 2]$
11. Prove : tangents to the curve  $y = x^2 - 5x + 6$  at points  $(2, )$  and  $(3, 0)$  are at right angles
12. Find the eq. Of normal to the curve  $x = a \cos^3 \theta, y = a \sin^3 \theta$  at  $\theta = \frac{\pi}{4}$
13. Find the points on curve  $9y^2 = x^3$  where normal to the curve makes equal intercepts with axes.
14. Find points on the curve  $xy + 4 = 0$  at which tangents to curve are inclined at an angle of  $45^\circ$  with  $x$  - axis
15. Find the equations of tangent & normal to the curves (i)  $y = a(1 + \cos \theta)$  at  $\theta = \frac{-\pi}{2}$  (ii)  $y = x^3 - x$  at  $x = 2$  (iii)  $x^2 + 3y + y^2 = 5$  at  $(1, 1)$
16. Prove that one  $\left(\frac{x}{9}\right)^n + \left(\frac{y}{9}\right)^n = 2$  touches the st. line  $\frac{x}{a} + \frac{y}{b} = 2$  for all  $n \in \mathbb{N}$  at  $(a, b)$
17. Show that curves  $xy = a^2$  and  $x^2 + y^2 = 2a^2$  touch each other.
18. Find interval in which  $f(x)$  is (i) increase (ii) decrease (a)  $f(x) = 2x^3 + px^2 + 12x + 20$  (ii)  $f(x) = x^4 = x^3/3$  (iii)  $f(x) = x^3 + \frac{1}{x^3}$
19. Separate  $(0, \frac{\pi}{2})$  in sub-intervals in which  $f(x) = \sin 3x$  is increasing or decreasing is also  $f(x) = \sin^4 x + \cos^4 x$ .
20. Find the intervals on which  $f(x) = 2x^3 - 3x^2 - 3(x+7)$  is (i) strictly increasing (ii) strictly decreasing.
21. Find local max & local min. If (i)  $f(x) = (\sin x - \cos x)$  where  $0 < x < \pi/2$  (ii)  $f(x) = \sin 4x + \cos 4x$  in  $(0, \pi/2)$
22. Show that  $\square$  of max area inscribed in a given circle is an equivalent D.
23. Show that semi-vertical angle of cone of max vol.  $K$  of given slant is  $\tan^{-1} \sqrt{2}$