CHAPTER - 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- A number of the form $z = a + ib$, where $a, b \in R$ is called a complex number.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- $a + ib = c + id$ if $a = c$, and $b = d$
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$

  but if $b, d = 0$ and $a > c$ then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- $-z = -a + i(-b)$ is called the Additive Inverse or negative of $z = a + ib$
- $\bar{z} = a - ib$ is called the conjugate of $z = a + ib$

$z^{-1} = \dfrac{1}{z} = \dfrac{a - ib}{a^2 + b^2} = \dfrac{\bar{z}}{|z|^2}$ is called the multiplicative Inverse of

$z = a + ib$ $(a \neq 0, b \neq 0)$

- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane
- Polar form of $z = a + ib$ is,

  $z = r(\cos\theta + i\sin\theta)$ where $r = \sqrt{a^2 + b^2} = |z|$ is called the modulus of z,

  θ is called the argument or amplitude of z.
- The value of θ such that, $-\pi < \theta \leq \pi$ is called the principle argument of z.
- $|z_1 + z_2| \leq |z_1| + |z_2|$
- $|z_1 z_2| = |z_1| . |z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \quad |z^n| = |z|^n, \quad |z| = |\bar{z}| = |-z| = |-\bar{z}|, \quad z\,\bar{z} = |z|^2$
- $|z_1 - z_2| \leq |z_1| + |z_2|$
- $|z_1 - z_2| \geq \left||z_1| - |z_2|\right|$
- For the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in R$, $a \neq 0$,

  if $b^2 - 4ac < 0$ then it will have complex roots given by,

  $$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Evaluate, $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

2. Evaluate, $i^{29} + \frac{1}{i^{29}}$

3. Find values of x and y if,

   $$(3x - 7) + 2iy = -5y + (5 + x)i$$

4. Express $\frac{i}{1+i}$ in the form a + ib

5. If $z = \frac{1}{3+4i}$, find the conjugate of z

6. Find the modulus of $z = 3 - 2i$

7. If z is a purely imaginary number and lies on the positive direction of y-axis then what is the argument of z?

8. Find the multiplicative inverse of $5 + 3i$

9. If $|z| = 4$ and argument of $z = \frac{5\pi}{6}$ then write z in the form $x + iy$; $x, y \in R$

10. If $z = 1 - i$, find $\text{Im}\left(\frac{1}{z\,\bar{z}}\right)$

11. Simplify $(-i)(3\ i)\left(\frac{-1\ i}{6}\right)^3$

12. Find the solution of the equation $x^2 + 5 = 0$ in complex numbers.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

13. For Complex numbers $z_1 = -1 + i$, $z_2 = 3 - 2i$

show that,

$$\text{Im}\ (z_1 z_2) = \text{Re}\ (z_1)\ \text{Im}(z_2) + \text{Im}\ (z_1)\ \text{Re}\ (z_2)$$

14. Convert the complex number $-3\sqrt{2} + 3\sqrt{2}\ i$ in polar form

15. If $x + iy = \sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

16. Find real value of θ such that,

$$\frac{1 + i\cos\theta}{1 - 2i\cos\theta} \text{ is a real number}$$

17. If $\left|\frac{z - 5i}{z + 5i}\right| = 1$, show that z is a real number.

18. If $(x + iy)^{\frac{1}{3}} = a + ib$, prove that, $\left(\frac{x}{a} + \frac{y}{b}\right) = 4(a^2 - b^2)$

19. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$

20. If $\left(\frac{2 + 2i}{2 - 2i}\right)^n = 1$, find the least positive integral value of n.

21. Find the modulus and argument of $z = 2 - 2i$

22. Solve the equation, $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

23. If z_1, z_2 are complex numbers such that, $\left|\frac{z_1 - 3z_2}{3 - z_1\bar{z}_2}\right| = 1$ and $|z_2| \neq 1$ then find $|z_1|$

24. Find the square root of $-3 + 4i$ and verify your answer.

25. If $x = -1 + i$ then find the value of $x^4 + 4x^3 + 4x^2 + 2$

ANSWERS

1. 0	2. 0
3. $x = -1, y = 2$	4. $\frac{1}{2} + \frac{1}{2}i$
5. $\bar{z} = \frac{3}{25} + \frac{4i}{25}$	6. $\sqrt{13}$
7. $\frac{\pi}{2}$	8. $\frac{5}{34} - \frac{3i}{34}$
9. $z = -2\sqrt{3} + 2i$	10. 0
11. $\frac{i}{72}$	12. $x = \pm i\sqrt{5}$

14. $z = 6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

16. $\theta = (2n+1)\frac{\pi}{2}, n \in z$

17. **Hint :** use property $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

19. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

20. n = 4

21. modulus = $2\sqrt{2}$, argument $= \frac{-\pi}{4}$

22. $x = \frac{\sqrt{2} \pm i\sqrt{34}}{2\sqrt{3}}$

23. **Hint :** use $|z|^2 = z.\bar{z}$, $|z_1| = 3$

24. $\pm (1 + 2i)$

25. 6