UNIT – III

LAWS OF MOTION AND FRICTION

KEY CONCEPTS

- According to the **principle of conservation of linear momentum,** the vector sum of linear momenta of all the bodies in an isolated system is conserved.

  Flight of rockets, jet planes, recoiling of a gun etc. are explained on the basis of this principle.

- Apparent weight of a man in an elevator is given by $W' = m(g \pm a)$.

  where mg = real weight of the man. Acceleration = (+a), when the lift is accelerating upwards and

  Acceleration = (–a) when the lift is accelerating downwards.

  when lift is *moving* in uniform motion then a = 0, $w' = mg$ = real weight.

  in free fall, $a = g \therefore W' = m(g - g) = 0$

  i.e. apparent weight becomes zero.

- When two bodies of masses m_1 and m_2 are tied at the ends of an inextensible string passing over a light frictionless pulley, **acceleration** of the system is given by

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} \text{ and}$$

  Tension in the string is $T = \dfrac{2m_1m_2g}{(m_1 + m_2)}$

- Impulse is defined as change in momentum.

Impulse $I = \vec{F}_{av} \times t = \vec{p}_2 - \vec{p}_1$

where t is the time for which average force acts, $\left(\vec{p_2} - \vec{p_1}\right)$ is change in linear momentum of the body.

- Any system is said to be in equilibrium if net force applied on the system is zero. In this case system is either at rest or in uniform motion.
- **Friction** is the opposing force that comes into play when one body is actually moving over the surface of another body or one body is trying to move over the surface of the other.

  Two causes of friction are : roughness of surfaces in contact; (ancient view)

  Force of adhesion between the molecules of the surfaces in contact. (Modern view)
- Type of solid friction :–

  1. Static friction. It comes in to effect when object is at rest but external force is applied.
  2. Dynamic friction- it comes in to effect when object is in motion.
  3. Rolling friction- it comes in to when object is in rolling.
- Limiting friction is the maximum value of static friction. Dynamic/Kinetic friction is somewhat less than the force of limiting friction.
- **Coefficient of friction** $\mu = F/R$

  when F = external force and R = normal reaction
- **Angle of friction** (θ) is the angle which resultant of *F* and *R* makes with the direction of *R*. The relation between μ and θ is $\mu = \tan \theta$
- **Angle of Repose** (a) is the minimum angle of inclination of a plane with the horizontal, such that a body placed on hte plane just begins to slide down. $\mu = \tan\alpha$
- **Centripetal force** is the force required to move a body uniformally in a circle.

- During motion on level curved road, the necessary centripetal force is provided by the force of friction between the tyres and the road. The maximum velocity with which a vehicle can go round a level curve without skidding is $v = \sqrt{\mu r g}$.

- To increase speed on turn, **curved roads are usually banked** *i.e.* outer edge of the curved road is raised suitably above the inner edge. If θ is the angle of banking, then

$$\tan\theta = \frac{v^2}{rg}$$

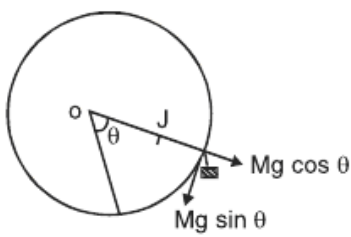
When frictional force is ignored, the optimum speed is $v_0 = (rg \tan\theta)^{1/2}$

- While rounding a banked curved road, maximum permissible speed is given by $v_{max} = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}\right]^{1/2}$ when friction is taken in to account.

- When a cyclist takes a turn. He bends a little inwards from his vertical position, while turning. Angle θ of bending from vertical position is given by

$$\tan\theta = \frac{v^2}{rg}$$

- **Motion along a vertical circle is a** non uniform circular motion. Tension in the string at any position is $T = \frac{mv^2}{r} + mg\cos\theta$, where θ is the angle of string with vertical line for looping with optimum speed :- (when tension at highest point is zero)



(i) velocity of projection at lowest point L is $v_L \geq \sqrt{5\,g\,r}$

(ii) velocity at the highest point H is $v_H \geq \sqrt{gr}$

(iii) Tension at lowest point $T_L \geq 6\ mg$

(iv) For oscillation in vertical circle $0 < v_L \leq \sqrt{2\,g\,r}$

(v) For leaving the vertical circle somewhere between

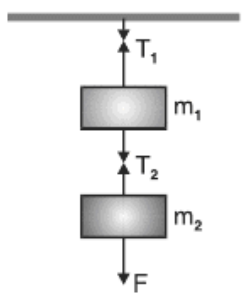
$90° < \theta < 180°, \sqrt{2gr} < v_L < \sqrt{5\,g\,r}$

- **Acceleration** of a body down a rough inclined plane $a = g(\sin\theta - \mu\cos\theta)$
- When a person of mass m climbs up a rope with acceleration a, the tension in the rope is $T = m(g + a)$

  When the person climbs down the rope with acceleration a, tension in the rope is $T = m(g - a)$
- When a body slides down a smooth inclined plane of inclination θ with the horizontal, its acceleration down the plane is $a = g\sin\theta$
- Suppose two masses m_1 and m_2 are suspended vertically from a rigid support, with the help of strings as shown in Fig. when mass m_2 is pulled down with a force F, then

  $T_2 = F + m_2 g$

  and $T_1 = F + (m_1 + m_2)\ g$

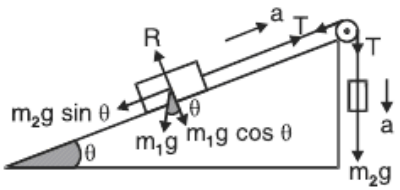


- When the same system of two masses attached to a string passes over a frictionless pulley at the edge of an inclined plane, as shown in Fig. equation wil be

$m_1 a = T - m_1 g \sin\theta$

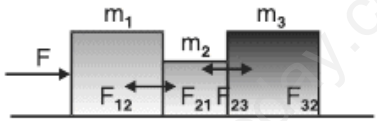
$m_2 a = m_2 g - T$

$R = m_1 g \cos\theta$



- **Three bodies in contact on a smooth horizontal table**

The action and reaction forces are as shown in Fig.



Common acceleration, $a = \dfrac{F}{m_1 + m_2 + m_3}$

Equation of motion of first body is

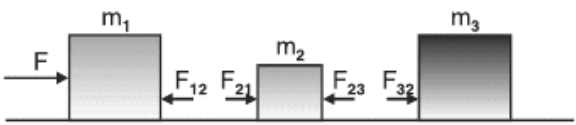
$F - F_{12} = m_1 a \qquad F_{12} = F_{21} = F_1$

Equation of motion of 2nd body is

$F_{21} - F_{23} = m_2 a \qquad F_{23} = F_{32} = F_2$

Equation of motion of 3rd body is

$F_{32} = m_3 a$

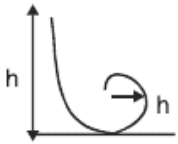


we get

$F_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$

$$F_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$$

- The minimum height through which a motor cyclist has to descend to a vertical loop of radius r is $h = \frac{5}{2}r$
- **Circular Motion**



- **Angular displacement (θ) :** The angular displacement of an object moving around a circular path is defined as the angle subtended by the radius vector at the centre of the circular path in the given time.
- **Angular velocity (ω) :** It is defined as the time rate of change of angular displacement of the object *i.e.* $\omega = d\theta/dt$. Its S.I unit is rad/s.
- **Uniform circular motion :** When a point object is moving on a circular path with a constant speed, then the motion of the object is said to be a uniform circular motion.
- **Centripetal acceleration :** It is defined as the acceleration of as object undergoing uniform circular motion. It always acts along the radius towards the centre of the circular path. The magnitude of centripetal acceleration is, $a = \frac{v^2}{r} = r\omega^2 = 4\pi^2 r\nu^2 = \frac{4\pi^2 r}{T^2}$
- The rate of change of angular velocity (ω) is called its angular acceleration (α) *i.e.*

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

- The acceleration which changes the magnitude of the velocity is called tangential acceleration. It is given by $a_T = r\alpha$, where α is the angular acceleration.

  The direction of tangential acceleration is along the tangent to curved path.

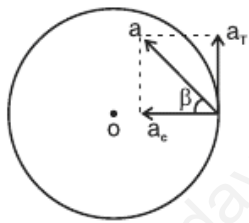
- When a body moves in a circular path with increasing angular velocity, it has two linear accelerations.

(i) centripetal acceleration, $a_c = v^2/r$; (ii) tangential acceleration, $a_T = r\alpha$;

Resultant acceleration of the body is

$$a = \sqrt{a_c^2 + a_T^2} \quad \text{and} \quad \tan\beta = \frac{a_T}{a_c}$$

where β is the angle between a and a_c



1 MARK QUESTIONS

1. A passenger sitting in a car at rest, pushes the car from within. The car doesn't move. Why?
2. Give the magnitude and directions of the net force acting on a rain drop falling with a constant velocity.
3. Why the passengers in a moving car are thrown outwards when it suddenly takes a turn?
4. What is the purpose of using shockers in a car?
5. Why are tyres made of rubber not of steel?
6. Wheels are made circular. Why?
7. A force is required to keep a body in uniform motion in a straight line. Comment.
8. On a rainy day skidding takes place along a curved path. Why?
9. Why does a gun recoils when a bullet is being fired?
10. Why is it difficult to catch a cricket ball than a tennis ball even when both are moving with the same velocity?
11. Calculate the impulse necessary to stop a 1500 kg car moving at a speed of 2 Sms^. (-37500 N-)
12. Lubricants are used between the two parts of a machine. Why?
13. What provides the centripetal force to a car taking a turn on a level road?

14. What is an inertial frame of reference?
15. An athlete runs a certain distance before taking a long jump. Why?
16. Action and reaction forces do not balance each other. Why?
17. The wheels of vehicles are provided with mudguards. Why?
18. China wares are wrapped in straw paper before packing?
20. Why is it difficult to walk on a sand?
21. The outer edge of a curved road is generally raised over the inner edge. Why?
22. Explain why the water doesn't fall even at the top of the circle when the bucket full of water is upside down rotating in a vertical circle?
23. Why does a speedy motor cyclist bends towards the centre of a circular path while taking a turn on it?
24. If the net force acting upon the particle is zero show that its linear momentum remains constant?

2 MARKS QUESTIONS

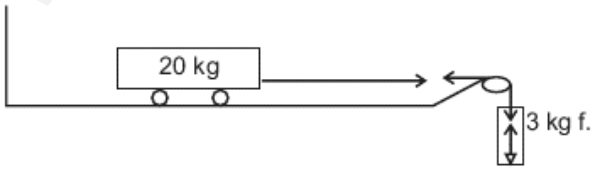
1. A man getting out of a moving bus runs in the same direction for a certain distance. Comment.
2. It is difficult to push a box full of clothes than an empty box. Explain.
3. The motion of a particle of mass m is described by h = ut + l/2gt2. Find the force acting on particle. (F = mg)
4. A particle of mass 0.3 kg is subjected to a force of F = -kx with k = 15 NrrT1. What will be its initial acceleration if it is released from a point 20 cm away from the origin? (a = -10 ms~2)
5. A 50 g bullet is fired from a 10 kg gun with a speed of 500 ms"1. What is the speed of the recoil of the gun. (2.5 ms"1)
6. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m by applying a force P at the free end of the rope. Find the force exerted by the rope on the block. F=M [P/ (M+m)
7. Three forces Fl5 F2 and F3 are acting on the particle of mass m which is stationary. F2 is perpendicular to Ft if F! is removed, what will be the acceleration of particle? (a = Fi/m)
8. A spring balance is attached to the ceiling of a lift. When the lift is at rest spring balance reads 49 N of a body hang on it. If the lift moves :-
   (i) Downward
   (ii) upward, with an acceleration of 5 ms"2
   (iii) with a constant velocity.

   What will be the reading of the balance in each case? (24N, 74 N, 49N)
9. It is easier to pull a roller than to push it. Why?

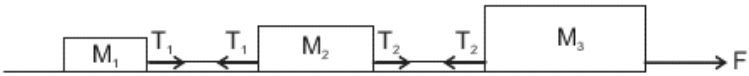
10. A horse can pull a cart and run in empty space. Why?
11. A bob of mass 0.1 kg hung from the ceiling of room by a string 2 m long is oscillating. At its mean position the speed of the bob is 1 ms^1. What is the trajectory of the oscillating bob if the string is cut when the bob is :-
    (i) At the mean position (Parabolic)
    (ii) At its extreme position (vertically downwards)
13. Define force of friction? How does ball bearing reduce friction?
14. Is larger surface area break on a bicycle wheel more effective than smaller surface area brake? Explain?

3 MARKS QUESTIONS

1. A block of mass 500g is at rest on a horizontal table. What steady force is required to give the block a velocity of 200 cm s'1 in 4 s?
2. A force of 98 N is just required to move a mass of 45 kg on a rough horizontal surface. Find the coefficient of friction and angle of friction? (0.22, 12°24')
3. Calculate the force required to move a train of 200 quintal up on an incline plane of 1 in 50 with an acceleration of 2ms"2. The force of friction per quintal is 0.5 N. (40200 N)
4. An aeroplane requires to take off a speed of 80 km h"1 on a runway of 100m. Mass of the plane is 10000 kg and coefficient of friction between the plane and the ground is 0.2. If the acceleration of the plane is uniform during take off, Calculate the minimum force required by the engine for the take off. (27. 13 N)
5. A smooth block is released from rest on a 30° incline and travels a distance d. If the time taken to slide on a rough 30° inclined surface is n times large to cover the same distance on a smooth incline. Find the coefficient of friction?
6. What is the acceleration of the block and trolly system as in fig., if the coefficient of kinetic friction between the trolly and the surface is 0.04? Also calculate tension in the string. Take g = 10 ms^{-2}, mass of string is negligible. (0.957 ms^{-2}, 27.13 N)



7. Three blocks of masses ml = 10 kg, m2 = 20 kg and m3 = 30 kg are connected by strings on smooth horizontal surface and pulled by a force of 60 N. Find the acceleration of the system and tensions in the string.



(T1=10N, T2=30N)

5 MARKS QUESTIONS

1. Define the principle of conservation of linear momentum. Deduce the law of conservation of linear momentum from Newton's third law of motion.
2. Why circular roads are banked? Derive an expression for angle of banking for safe circular turn?
3. Obtain an expression for minimum velocity of projection of a body at the lowest point for Looping a vertical loop.
4. Show that the area under the force-time graph gives the magnitude of the impulse of the given force for the following case when (i) force is constant (ii) variable force.
5. Derive an expression for acceleration of a body down a rough inclined plane? (Sliding only)